Volume 1 No. 2, August 2011
International Journal of Science and Technology

http://www.ejournalofsciences.org

Multi-Grade Economic Trainee Batch – Size Manpower Model with varying cost function

[1]M. Jeeva , [2]V. Nalina
[1]Department of Mathematics, Ethiraj College for Women,
Chennai 600 008, INDIA
[2] Department of Mathematics, A.M.Jain College,
Chennai 600 114, INDIA

ABSTRACT

A multi-grade deterministic-trainee inventory model with a constant demand is considered in this paper. A Manpower inventory model where in the trainees are trained at a constant rate and are inspected at regular intervals of time. Inefficient persons are identified and retrained to bring them to the expected level of efficiency. The single trainee cost is assumed to be a function of trainee rate. We formulate a generalized single trainee cost function, which brings flexibility in deciding the number of persons to be trained over a training period. The objective of this paper is to obtain Mult-Grade Economic Trainee batch-size that minimizes the total cost function. The model solution is justified through a numerical example.

Keywords: *Multi-Grade Trainee Inventory Model, Trainee Rate, Trainee Cost, Manpower Inventory Model.*

I. INTRODUCTION

The classical Economic Production Quantity (EPQ) Model has been in use for a long time in research field. It can be considered as an extension of Economic Order Quantity (EOQ) Model. In classical Economic Production Lot Size (EPL) Model the production rate of a machine is assumed to be pre determined and inflexible Hax[7]. However the variations in the machine production rate was proposed by Schweitzer et al[15]. Alder and Nanda[1,2] extended the EPL Model to both single production and multi production cases to situations where learning effects cause the production rate to increase. Cheng[5] proposed an extension to the EPL Model in which demand exceed supply and the production process is imperfect. The EPL Model under volume flexibility was presented by Moutaz[13]. Lot sizing capacity utilization in a production process with defective item, process correction and rework was given by Lee[11]. Bhandari et al.[4] have developed an extension of EPQ in which production flexibility of manufacturing system is proposed. Lee et al[12] developed a model of batch quantity in a multi-state production system considering various proportions of defective items produced in every state while they ignored the rework situation. Gupta and Chakrabarthy [6] considered an imperfect multi-stage manufacturing system wherein the defectives are collected and reworked.[8]Jamal *et al.*considered a single production system with rework option incorporating two cases of rework process. In the first case they considered the rework is done within the same cycle where the item is produced. In the second case the defective items are accumulated upto N cycles and accumulated items are reworked after the Nth cycle. They developed optimal batch quantity models to minimize the system cost.

This research deals with manpower inventory model wherein we consider human resource as inventory for an organization. Here we consider a training consultancy (organization) that selects trainees and absorbs persons after they complete the training process successfully. Inspection is done after every regular interval of time and inefficient persons are identified and are retrained to make them efficient. The retraining is done within the training cycle. Jeeva & Nalina [9] have studied an economic trainee batch – size model with variable cost function and where the model gives the flexibility in deciding the number of persons to be trained over a training period.

Our main concern in this study is to find optimal trainee batch size and optimal trainee rate for a multi grade system that minimizes the total cost. In this paper, a formulation of the problem is developed and optimization techniques are performed to obtain the solution. A numerical example is solved to illustrate the model.

II. MODEL ANALYSIS

In this research, the inventory model is developed for the operational policy in which we retrain the persons whenever they fail in their efficiency level and it is assumed that all the persons are brought up to the expected level of efficiency once retraining is completed. As retraining cost is comparatively less than training a fresh candidate, the organization prefers and believes in retraining.

In this policy, we consider an organization where persons are selected through interview and trained for their requirements and absorbed or shifted to the permanent vacancy. During the training period inspection is carried over frequently to identify their level of efficiency. Each cycle time is divided into two parts. In the first part training takes place while in the later part only absorption takes place.

Volume 1 No. 2, August 2011
International Journal of Science and Technology

http://www.ejournalofsciences.org

Whenever the inefficient persons are identified, they are retrained and brought to the expected level of efficiency. We assume that the inefficient persons are identified at the rate of α_i during training time. The replenishment rate during training time is [k_i(1- α_i) – R_i] and number of inefficient persons is $\alpha_i Q_i$ for grade i. These inefficient persons are retrained at the rate of k_i persons per unit time. Hence, inventory gets built up during retraining time. After which training stops and only absorption takes place. During absorption time all the trained persons are absorbed by the organization.

A multi grade deterministic inventory manpower problem with a constant demand rate R_i is considered. In this policy an attempt is made to study training, retraining and absorbing policy in multi grade training system. We consider a training consultancy where in persons are selected trained according to the job requirements of organizations. It is more sensible to consider multi grade of training in a manpower system because all the persons selected for training need not be of the same efficiency level. So, the persons selected are first segregated according to their efficiency. This is done through a scrutiny or a screening test the cost of which is assumed to be negligible. Then they are segregated into n training grades. To start with persons in grade1 are with very less efficiency than persons in grade 2 and so on. Consequently persons in grade 1 require more training compared to persons in grade 2 and so on. Figure 1 gives the description of the model.

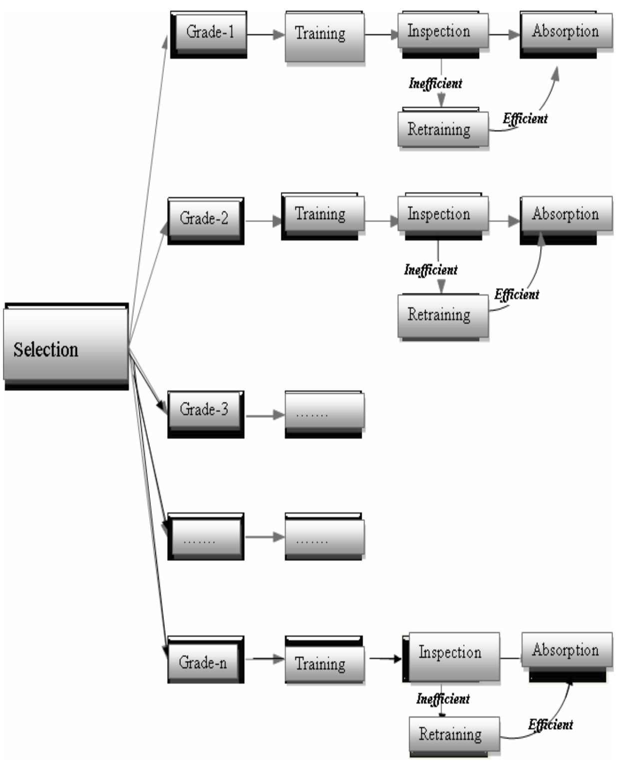


Figure 1

III. ASSUMPTIONS AND NOTATIONS

In this research, the inventory model is developed for the operational policy in which we retrain the persons whenever they fail in their efficiency level and it is assumed that all the persons are brought up to the expected level of efficiency once retraining is completed.
The assumptions and notations used in this model are:

Assumptions:

- A multi grade deterministic inventory system is considered
- Trainee batch size (Q_i)and trainee rate (k_i) are decision variables
- Trainee rate is greater than demand rate
- In each cycle the proportion of in efficient person is constant
- Shortages are not allowed
- Single trainee cost is a function of trainee rate
- In efficient persons are trained to reach their expected efficiency level

Inspection cost is included as inspection takes place frequently to identify the inefficient persons and retrained them.

Notations:

1. k_i –Training rate, the number of persons getting trained in grade i per year (i=1,2,3,…..n).
2. Q_i – number of persons trained for the ith grade(i=1,2,3,…..n).
3. R_i – Demand rate for the ith grade (demand for trained persons for grade i per year)
4. S_i – Set up cost for grade i (for i=1,2,3,…..n).
5. H_i - Holding cost per person per year in gradel i.
6. T- Cycle time
7. TC – Total cost
8. α_i – Proportion of inefficient persons identified in each grade i.
9. I_i - Inspection cost for the grade i (for i=1,2,3,…..n).

10. B_i -bench cost paid for the reserve employees in grade i(for i=1,2,3,…..n).

11. L_i – Labor cost for grade-i per cycle

12. F – Positive constant

13. E(k_i) = $\frac{L_i}{k_i}$ + F k_i , E(k_i) – Single trainee cost function as a function of trainee rate

IV. MODEL FORMULATION

The objective of this model is to minimize the relevant total cost. We assume that the inefficient persons of each grade is identified at the rate of α_i (0< α_i <1) during training time. The total replenishment rate during training time is [k_i (1- α_i) – R_i] and total number of inefficient persons of each grade are $\alpha_i\, Q_i$. These inefficient persons are retained at the rate of k_i persons per unit time. Hence, inventory gets built up during retraining time. During absorption time all the trained persons are absorbed by the organization.

Set up cost : Every training grade needs the prerequisite of arrangement of place for training etc. Hence the setup cost per unit time is equal to:

$$\text{Set up cost } = \sum_{i=1}^{n} \frac{S_i R_i}{Q_i} \qquad (1)$$

Inventory Holding cost:

During the training period (inventory) persons are trained at the rate of k_i persons per unit time which is greater than the demand rate R_i . Thus the organization has to bear the cost of retaining them until they are absorbed by the organization. Hence the retaining cost is given by,

$$\text{Inventory holding cost} = \sum_{i=1}^{n} H_i \bar{i}$$

Where i is the inventory level and $\bar{i} = \frac{i}{T}$ gives the average inventory.
Total inventory for the i[th] batch is,

$$i = \sum_{i=1}^{n} \frac{1}{2} \left[\frac{Q_i}{k_i}\right] \{ -\alpha_i^2 Q_i + k_i \frac{Q_i}{R_i} - Q_i - \alpha_i Q_i \}$$

Hence the average inventory, $\bar{i} = \frac{i}{T} = \frac{R_i}{Q_i}\, i$

$$\text{Average inventory} = \frac{Q_i}{2} \{1 - \frac{R_i}{k_i} (1+\alpha_i+\alpha_i^2)\}$$

Inventory holding cost= $H_i\, \bar{i}$

$$= \sum_{i=1}^{n} H_i \frac{Q_i}{2} \{1 - \frac{R_i}{k_i}(1+\alpha_i+\alpha_i^2)\} \qquad (2)$$

Training cost:

Training cost in each cycle is given by

$$\text{Training cost} = \sum_{i=1}^{n} E(k_i)\, R_i \qquad (3)$$

Inspection cost:

During the training period Inspection takes place frequently to identify the inefficient persons. Hence the inspection cost per unit time is given by:

$$\text{Inspection cost} = \sum_{i=1}^{n} I_i\, R_i \qquad (4)$$

Retraining cost:

Whenever inefficient persons are identified they are given extra training in order to bring them to their expected grade of efficiency.

$$\text{Retraining cost} = \sum_{i=1}^{n} E(k_i)\ \alpha_i R_i \qquad (5)$$

Reserve employees holding cost:

Since we assume that inefficient persons are identified at the rate of α_i of trained persons in each level, in order to maintain the batch size $\alpha_i Q_i$ employees are maintained as reserve employees.

$$\text{Reserve employees holding cost} = \sum_{i=1}^{n} B_i \alpha_i Q_i \qquad (6)$$

Total cost:

The total cost per cycle can be expressed as, sum of all the cost from equations (1) to (6)

$$TC\ (Q_1, Q_2, Q_3, \ldots\ldots Q_n,\ k_1, k_2, k_3, \ldots\ldots k_n) = \sum_{i=1}^{n}\frac{S_iR_i}{Q_i} + \sum_{i=1}^{n}H_i\frac{Q_i}{2}\{1-\frac{R_i}{k_i}(1+\alpha_i+\alpha_i^{\,2})\} + \sum_{i=1}^{n}[\frac{L_i}{k_i}+Fk_i]R_i(1+\alpha_i) + \sum_{i=1}^{n}I_i R_i + \sum_{i=1}^{n}\alpha_iB_iQ_i \quad (7)$$

Thus a non-linear programming problem is formulated as follows:

$$\text{Minimize TC} = \sum_{i=1}^{n}\frac{S_iR_i}{Q_i} + \sum_{i=1}^{n}H_i\frac{Q_i}{2}\{1-\frac{R_i}{k_i}(1+\alpha_i+\alpha_i^{\,2})\} + \sum_{i=1}^{n}[\frac{L_i}{k_i}+Fk_i]R_i(1+\alpha_i) + \sum_{i=1}^{n}I_i R_i + \sum_{i=1}^{n}B_i\alpha_iQ_i \quad (8)$$

Subject to the constraints:

$k_i \leq R_i$ for(i=1,2,3,…….n) (9)

and

Q_i and k_i must be integers. (10)

V. SOLUTION PROCEDURE

MGTVC (Multi-Grade Trainees with Variable Cost function) problem is discussed. The main objective of this model is to determine the optimal batch size(Q_i,)and optimal trainee rate(k_i) for each grade so as to minimize the total cost. The total cost is a function of multi-variables Q_i and k_i.

$$TC\ (Q_1, Q_2, Q_3, \ldots\ldots Q_n,\ k_1, k_2, k_3, \ldots\ldots k_n) =$$

$$\sum_{i=1}^{n}\frac{S_iR_i}{Q_i} + \sum_{i=1}^{n}H_i\frac{Q_i}{2}\{1-\frac{R_i}{k_i}(1+\alpha_i+\alpha_i^{\,2})\} +$$

$$\sum_{i=1}^{n}[\frac{L_i}{k_i}+Fk_i]R_i(1+\alpha_i) + \sum_{i=1}^{n}I_i R_i + \sum_{i=1}^{n}\alpha_iB_iQ_i \quad (11)$$

The partial derivatives with respect to the trainee rate and trainee batch size are set to zero,

$$\frac{\partial TC}{\partial Q_i}=0,\ \frac{\partial TC}{\partial k_i}=0 \quad (12)$$

And the optimal solutions are evaluated. Conditions (12) can be explicitly written as:

$$\frac{\partial TC}{\partial Q_i} = -\frac{S_iR_i}{Q_i^2} + \frac{H_i}{2}\{1-\frac{R_i}{k_i}(1+\alpha_i+\alpha_i^{\,2})\} + \alpha_iB_i = 0 \quad (13)$$

Gives,

$$Q_i^2 = \frac{2R_iS_ik_i}{H_i[k_i - R_i(1+\alpha_i+\alpha_i^2)] + 2k_i\alpha_iB_i} \quad (14)$$

and

$$\frac{\partial TC}{\partial k_i} = \frac{H_iQ_i}{2}\{\frac{R_i}{k_i^2}(1+\alpha_i+\alpha_i^{\,2})\} + [[-\frac{L_i}{k_i^2}+F]R_i(1+\alpha_i) = 0 \quad (15)$$

$$\text{Gives, } k_i^2 = \frac{2L_i(1+\alpha_i) - H_iQ_i(1+\alpha_i+\alpha_i^2)}{2F(1+\alpha_i)} \quad (16)$$

Substituting these values in equation (8) we get minimum TC(Q_i, k_i).

Algorithm-1: To solve MGTVC problem

The algorithm to solve the optimal batch size and optimal trainee rate are as follows:

Step 0: Initialize and store S_i, H_i, R_i, α_i, L_i, F, B_i and I_i

Step 1: Compute the optimal batch size(Q_i) and the optimal trainee rate (k_i)by solving the simultaneous equations

$$Q_i = \sqrt{\frac{2R_iS_ik_i}{H_i[k_i - R_i(1+\alpha_i+\alpha_i^2)] + 2k_i\alpha_iB_i}} \quad \text{and}$$

$$k_i = \sqrt{\frac{2L_i(1+\alpha_i) - H_iQ_i(1+\alpha_i+\alpha_i^2)}{2F(1+\alpha_i)}}$$

Step 2: If Q_i and k_i are integers then go to Step 4. Else continue with step 3.

Step 3: If Q_i and k_i are not integers then find the combination of integer approximation of Q_i^* and k_i^* that minimizes TC by using partition Bound technique.

Step 4: Compute $TC_{\min}$ $(Q_1^*, Q_2^*, Q_3^*, ...Q_n^*, k_1^*, k_2^*, k_3^*, ...k_n^*)$

Stop.

In order to obtain the optimal solution we need to verify the convexity of the objective function

$$TC_{\min}\ (Q_1^*, Q_2^*, Q_3^*, ...Q_n^*,\ k_1^*, k_2^*, k_3^*, ...k_n^*).$$

The pair $(Q_1^*, Q_2^*, Q_3^*, ...Q_n^*,\ k_1^*, k_2^*, k_3^*, ...k_n^*)$ will be an optimal solution, as the associated Hessian matrix H_1 (X) is positive definite (see Appendix A).

Algorithm-2: For integer approximation

For obtaining the optimal integer solution for the MGTVC problem we apply the partition bound algorithm (partition bound technique and manpower model, chapter IV, in the Ph.D thesis of Kala [10])

Step 1: Solve the given MGTVC problem and obtain the optimal solution ignoring the integer restriction on the variables. If the all variables are integers then the optimal integer solution is obtained. If not go to step 2.

Step 2: Let $(Q_1, Q_2, Q_3, ...Q_n,\ k_1, k_2, k_3, ...k_n)$ be the solution of the given problem. If one or more of the values of Q_i or k_i, i = 1, 2, 3, ……. n, are non-integer values then choose one of $Q_1, Q_2, Q_3, ...Q_n$, $k_1, k_2, k_3, ...k_n$ with the greatest fractional part. Let l.b. ≤ Q_i ≤ u.b., where l.b. is the lower bound of integral part of Q_i and u.b. is the upper bound of integral part of Q_i .

Step 3: Divide the given problem into two sub problems with the help of lower bound and upper bound. We introduce additional constraint as Q_i = $[Q_i]$ to one sub problem and Q_i = $[Q_i]$ +1 to the second sub problem, with other values of Q's and k's unchanged.

Step 4: In the sub problem 1 replace Q_i = $[Q_i]$ and the problem reduces to MGTVC and in the second sub problem replace Q_i by $[Q_i]$ +1. Both the problems do not have Q_i as variables.

Step 5: Choose the variable Q_j with the greatest fractional part among the remaining (n-1) variables $Q_1, Q_2, Q_3, ...Q_{j-1}, Q_{j+1}, ..Q_n$ of step 2 and apply steps 2,3 and 4 which will yield 4(n-1) sub problems.

Step 6: Obtain TC for each sub problem and among the TC values choose the sub problem which has minimal TC value for which Q_i is a fixed integer.

Step 7: Each of these minimal sub problem will yield 2 (2 n-1) solutions. Choose the solution which has the minimal TC.

Step 8: This procedure is continued until all the Q_i 's and k_i's in the solution are integers.

Step 9: Among the integer solutions the one which optimizes (minimize) the objective function is the optimal integer solution of the given MGTVC problem.

Stop.

Proposition 1:

For 0 < α_i < 1, the optimal value k_i^* of k_i satisfies the inequality $L_i \geq \frac{H_iQ_i}{2}$.

Proof: using the expression in (14), we see that

$$2L_i(1+\alpha_i) - H_iQ_i(1+\alpha_i+\alpha_i^2) \geq 0$$

Hence,

$$L_i \geq \frac{H_iQ_i(1+\alpha_i+\alpha_i^2)}{2(1+\alpha_i)}$$

$$L_i \geq \frac{H_iQ_i(1+\alpha_i+\alpha_i^2)(1+\alpha_i)^{-1}}{2}$$

$$L_i \geq \frac{H_iQ_i}{2}(1+\alpha_i+\alpha_i^2)(1-\alpha_i+\frac{\alpha_i^2}{2}-....)$$

$$L_i \geq \frac{H_iQ_i}{2}(1-\frac{\alpha_i^2}{2}....)$$

As α_i lies between zero and one we neglect higher powers of it.
gives ,

$$L_i \geq \frac{H_iQ_i}{2}$$

Proposition 2:

$\frac{k_i}{R_i} \geq (1+\alpha_i+\alpha_i^2)$ is the condition for existence of optimal batch size (Q_i^*).

Proof:

RHS of equation (12) is positive. As the numerator is positive the denominator should also be positive. Hence,

$$k_i > R_i\ (1+\alpha_i+\alpha_i^2)$$

So: $\frac{k_i}{R_i} \geq (1+\alpha_i+\alpha_i^2)$.

Corollary:

For α_i in the range (0,1), it follows that

$\frac{k_i}{R_i}$ must be greater than or equal to 3.

Proposition 3:

The optimal decision rule for Q_i^* is given by,

$$(Q_i^*\text{-}1)\ Q_i^* \leq \frac{2R_iS_ik_i}{H_i[k_i-R_i(1+\alpha_i+\alpha_i^2)]+2k_i\alpha_iB_i}$$

$$\leq (Q_i^*+1)\ Q_i^* \qquad (17)$$

Where $Q_i^* = \sqrt{\frac{2R_iS_ik_i}{H_i[k_i-R_i(1+\alpha_i+\alpha_i^2)]+2k_i\alpha_iB_i}}$

Special cases:

(i) The Economic Treinee Batch – Size Manpower model with variable cost function
In this case taking n=1 in (17) we get,

$$(Q_i^*\text{-}1)\ Q_i^* \leq \frac{2RSk}{H[k-R(1+\alpha+\alpha^2)]+2k\alpha B} \leq (Q_i^*+1)\ Q_i^*$$

which agrees with the M. Jeeva & V. Nalina[9].

(ii) The EOQ problem with single rate of production

In this case taking n=1 and α =0 in (17) we get,

$$(Q_i^*\text{-}1)\ Q_i^* \leq \frac{2RSk}{H[k-R]} \leq (Q_i^*+1)\ Q_i^*$$

which agrees with Bartmann and Beckmann[3].

(iii) The fundamental EOQ problem
In this case taking n=1, α =0 and $k=\infty$ in (17) we get,

$$(Q_i^*\text{-}1)\ Q_i^* \leq \frac{2RS}{H} \leq (Q_i^*+1)\ Q_i^*$$

which agrees with the Sivazlian [14] formula.

The following numerical example will illustrate the model and also the validity of the propositions are checked.

Numerical example:

To illustrate the usefulness of the model developed, consider a training consultancy with two grades of training (n=2)where in the parameters are given in the table and the values of the variables Q_1, Q_2, k_1, k_2 are calculated using the algorithm-1:

TABLE 1:

$R_{i(persons)}$	H_i(Rs)	S_i(Rs)	α_i	L_i(Rs)	F	B_i(Rs)	I_i(Rs)	Q_i (persons)	k_i (persons)
100	2000	10000	0.02	50000	0.1	3000	3000	47.1923	167.008
80	2200	10200	0.01	50100	0.1	3300	3200	44.1322	124.488

And minimum total cost is,

TC (Q_1, Q_2, k_1, k_2)= Rs.701124/year.

It is necessary that Q_i and k_i must be integers, as it specifies the number of persons. So, we try to find the best combination of integer values that will minimize the total cost.

This can be achieved by using partition Bound technique. For obtaining the optimal integer solution for the MGTVC problem we apply the partition bound algorithm [10].

Step 1: Solve the given MGTVC problem and obtain the optimal solution ignoring the integer restriction on the variables.
The (Q_1, Q_2, k_1, k_2) variables are (47.1923, 44.1322, 167.008, 124.488) not integers, go to step 2.

Step 2: Since Q_i or k_i, i = 1, 2 are non-integer values then choose one of Q_1, Q_2, k_1, k_2 with the greatest fractional part.
Here 47 ≤ Q_1 ≤ 48.

Step 3: Divide the given problem into two sub problems with the help of lower bound and upper bound. We introduce additional constraint as Q_1 = 47 to one sub problem and Q_1=48 to the second sub problem for detailed results see figure-2.

Step 4: In the sub problem 1 replace Q_1 = 47and the problem reduces to MGTVC problem without the variable Q_1.
And in the second sub problem replace Q_1 by 48.

Step 5: Choose the variable Q_2 with the greatest fractional part among the remaining 3 variables Q_2, k_1, k_2 of step 2 and apply steps 2,3 and 4. By using the above procedure we obtain 8 sub problems.

Step 6: Obtain TC for each of the sub problems, among these we obtain 5 sub problems with same minimal TC value.

Step 7: Each of these 5 minimal sub problems yield 6 more sub problems with 3 variables. Thus yields 30 sub problems.

Step 8: Among the 30 sub problems only 2 sub problems have same minimal TC value. Now we have 2 variables to be integerized.

Step 9: It is noted that both sub problems 21 and 12 have same values for Q_1 and k_1. Hence it gives identical problem. Now this yields 4 more sub problems.

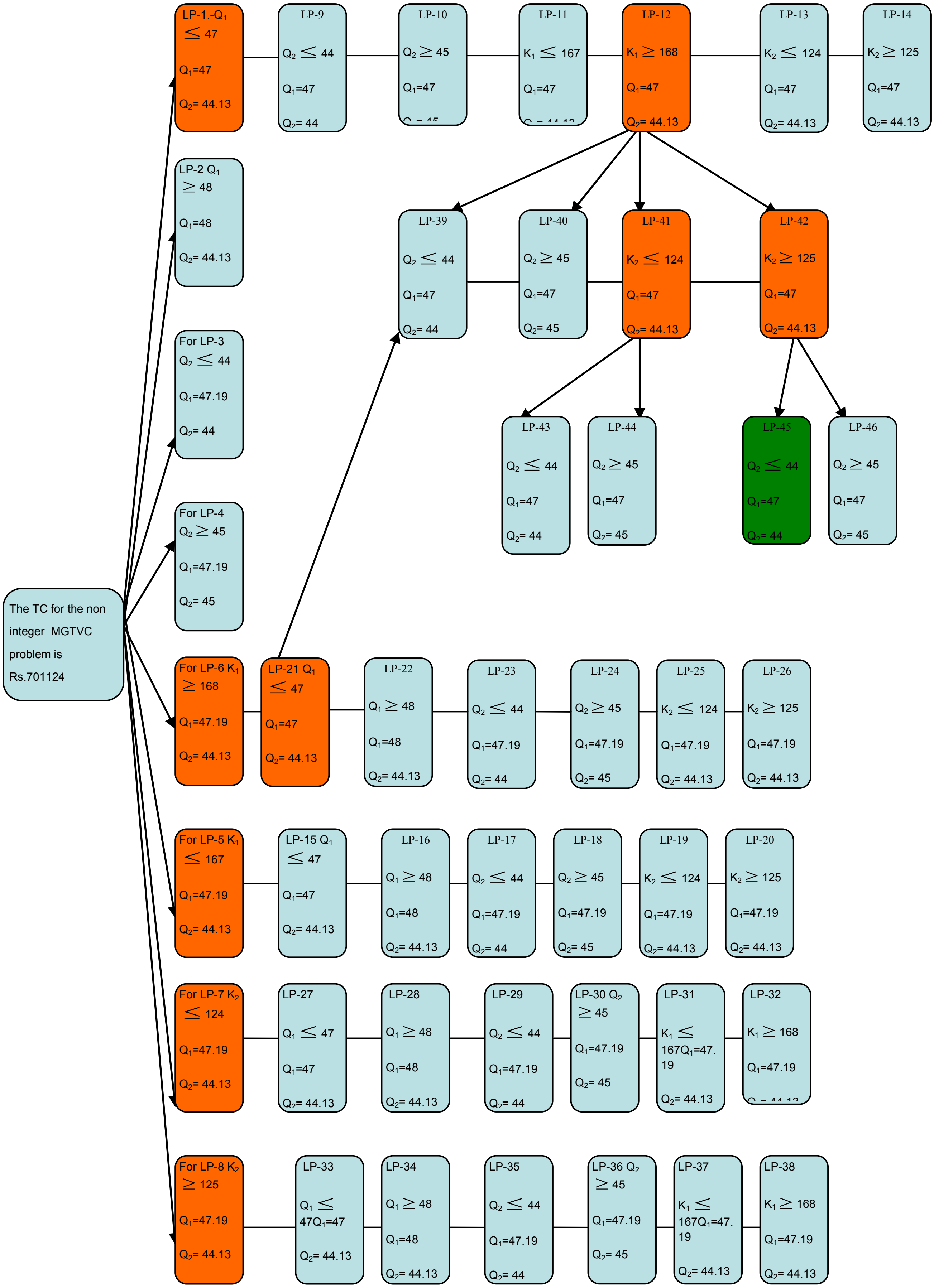
Step 10: Out of these 4 sub problems 2 sub problem have same TC value with 2 variables.

Step 11: Sub problems 41 and 42 yield 2 +2 sub problems, out of which we can obtain one optimal solution with all integer values. This is given by

TC_{min} ($Q_1^*, Q_2^*, k_1^*, k_2^*$) = Rs.701123/year with Q_1 = 47, k_1=168, Q_2 =44, k_2=125.

Stop.

This numerical example illustrates the proposed model. Also we have obtained an integer values for all variables.



The TC for the non integer MGTVC problem is Rs.701124
LP-1.-Q1 ≤ 47 Q1=47 Q2= 44.13
LP-9 Q2 ≤ 44 Q1=47 Q2= 44
LP-10 Q2 ≥ 45 Q1=47
LP-11 K1 ≤ 167 Q1=47
LP-12 K1 ≥ 168 Q1=47 Q2= 44.13
LP-13 K2 ≤ 124 Q1=47 Q2= 44.13
LP-14 K2 ≥ 125 Q1=47 Q2= 44.13
LP-2 Q1 ≥ 48 Q1=48 Q2= 44.13
For LP-3 Q2 ≤ 44 Q1=47.19 Q2= 44
For LP-4 Q2 ≥ 45 Q1=47.19 Q2= 45
LP-39 Q2 ≤ 44 Q1=47 Q2= 44
LP-40 Q2 ≥ 45 Q1=47 Q2= 45
LP-41 K2 ≤ 124 Q1=47 Q2= 44.13
LP-42 K2 ≥ 125 Q1=47 Q2= 44.13
LP-43 Q2 ≤ 44 Q1=47 Q2= 44
LP-44 Q2 ≥ 45 Q1=47 Q2= 45
LP-45 Q2 ≤ 44 Q1=47 Q2= 44
LP-46 Q2 ≥ 45 Q1=47 Q2= 45
For LP-6 K1 ≥ 168 Q1=47.19 Q2= 44.13
LP-21 Q1 ≤ 47 Q1=47 Q2= 44.13
LP-22 Q1 ≥ 48 Q1=48 Q2= 44.13
LP-23 Q2 ≤ 44 Q1=47.19 Q2= 44
LP-24 Q2 ≥ 45 Q1=47.19 Q2= 45
LP-25 K2 ≤ 124 Q1=47.19 Q2= 44.13
LP-26 K2 ≥ 125 Q1=47.19 Q2= 44.13
For LP-5 K1 ≤ 167 Q1=47.19 Q2= 44.13
LP-15 Q1 ≤ 47 Q1=47 Q2= 44.13
LP-16 Q1 ≥ 48 Q1=48 Q2= 44.13
LP-17 Q2 ≤ 44 Q1=47.19 Q2= 44
LP-18 Q2 ≥ 45 Q1=47.19 Q2= 45
LP-19 K2 ≤ 124 Q1=47.19 Q2= 44.13
LP-20 K2 ≥ 125 Q1=47.19 Q2= 44.13
For LP-7 K2 ≤ 124 Q1=47.19 Q2= 44.13
LP-27 Q1 ≤ 47 Q1=47 Q2= 44.13
LP-28 Q1 ≥ 48 Q1=48 Q2= 44.13
LP-29 Q2 ≤ 44 Q1=47.19 Q2= 44
LP-30 Q2 ≥ 45 Q1=47.19 Q2= 45
LP-31 K1 ≤ 167Q1=47.19 Q2= 44.13
LP-32 K1 ≥ 168 Q1=47.19
For LP-8 K2 ≥ 125 Q1=47.19 Q2= 44.13
LP-33 Q1 ≤ 47Q1=47 Q2= 44.13
LP-34 Q1 ≥ 48 Q1=48 Q2= 44.13
LP-35 Q2 ≤ 44 Q1=47.19 Q2= 44
LP-36 Q2 ≥ 45 Q1=47.19 Q2= 45
LP-37 K1 ≤ 167Q1=47.19 Q2= 44.13
LP-38 K1 ≥ 168 Q1=47.19 Q2= 44.13

VI. CONCLUSION

In this paper, an inventory Manpower model is being proposed for a multi-grade trainee system in which the trainee rate is taken as the variable. The multi-grade trainee cost function is taken as the function of trainee rate. The study not only brings flexibility in deciding the number of persons to be trained for each grade over a training period but also demonstrates the importance of taking in to account the quality of the trained persons. A simple algorithm is developed to compute optimal batch size and trainee rate. Economic Trainee batch-size and optimal trainee rate that minimizes the total cost We have been obtained.

REFERENCES

[1] ALDER. G.L. AND NANDA. R. (1974) The effects of learning an optimal lot size determination-single product case AIIE Trans. 6, PP. 14-20

[2] ALDER. G.L. AND NANDA. R. (1981) The effects of learning an optimal lot size determination-multiple product case AIIE Trans. 13, PP. 91-95

[3] BARTMANN D. and BECKMANN M.J.(1992) Inventory control. Models and Methods, Lecture Notes in Economic and Mathematical systems, Springer Verlag.

[4] BHANDARI, R.M. and P.K. SHARMA (1999) The economic production lot-size model with variable cost function, Opsearch, Vol.36. No.2 PP.137-150

[5] CHENG, T.C.E.(1991) An Economic order quantity model with demand dependent unit production cost and imperfect production process. AIIE Trans. 23(1) PP. 23-28.

[6] GUPTA, T. and CHAKRABARTHY, S., Looping in a multi-stage production system. Int.J.Prod. Res, (1984), 22, 299-301.

[7] HAX A.C. AND CONDEA. D (1984) Production and inventory management. Prentice-Hall, Englewood Cliff, N.J.

[8] JAMAL, A.M.M., SARKER, B.R., and Mondal, S., (2004), Optimal manufacturing batch size with rework process at a single stage production system, Computers and Industrial Engineering, 47, 77-89.

[9] JEEVA M and NALINA V (2011) The Economic Trainee Batch – size manpower model with variable cost function, Vol. 2, Issue.6, PP,185-190, June, 2011.

[10] KALA. S (2009) Stochastic Fractional and Integer programming models with reference to manpower Planning, Ph.D thesis, 66-83.

[11] LEE H.L. (1992) Lot sizing to reduce capacity utilization in a production process unit with defective items process correction rework Mgmt Sci. 9, PP. 1314-1328

[12] LEE, H.H., CHANDRA, M.J. and DELEVEAUX, V.J., (1997), Optimal batch size and investment in multistage production system with scrap. Production planning and control, 8(6), 586-596.

[13] MOUTAZ KHOUZA (1995) The economic production lot size model under volume flexibility comp OPS Res. Vol. 22, No.5 PP.515-525

[14] SIVAZLIAN B.D.(1974), A continuous –Review (s , S) Inventory system with Arbitrary Interarrival distribution between Unit Demand, Operation Research, 22,1974,65-71.

[15] SCHWEITZER. P.J. AND SEIDMAAN. A. (1991) Optimizing processing rates for flexible manufacturing system, Mgmt. Sci. 37.

APPENDIX

PROOF OF CONVEXITY OF A MULTI-GRADE TRAINEE VARIABLE COST FUNCTION

It is necessary to prove convexity of a multi-variable function for proving the solution of the objective function to be minimum. For proving this we need to find the associated Hessian matrix H(X), and it should be positive definite for all values of X in **R.**

The Hessian matrix associated with TC ($Q_1, Q_2, Q_3, \ldots\ldots Q_n, k_1, k_2, k_3, \ldots\ldots k_n$) is given by H_1. From equations (13) and (15), the second derivatives are calculated as follows,

$$\frac{\partial^2 TC}{\partial Q_i^2} = \frac{2S_i R_i}{Q_i^3} > 0 \text{ for i = 1, 2, 3……n} \quad (18)$$

Since S_i, R_i and Q_i are always positive.

International Journal of Science and Technology

http://www.ejournalofsciences.org

$\frac{\partial^2 TC}{\partial Q_i Q_j}$ = 0, where i $\neq$ j. since Q_i and Q_j are batch sizes of different grades.

for i = 1, 2, 3……n and for j = 1, 2, 3……n (19)

$\frac{\partial^2 TC}{\partial^2 k_i^2} = \frac{H_i Q_i}{2}$ [- $\frac{R_i}{k_i^3}(1+\alpha_i+\alpha_i^2)$] + [$\frac{2L}{k_i^3}$ + F] $R_i(1+\alpha_i)$, for i = 1, 2, 3……n (20)

From Proposition 1 we infer that the above second order derivative in equation (20) is positive.

$\frac{\partial^2 TC}{\partial k_i k_j}$ = 0, where i $\neq$ j , since k_i and k_j are trainee rates of different grades. (21)

$\frac{\partial^2 TC}{\partial Q_i k_j}$ =0, where i $\neq$ j , since Q_i and k_j are the batch size and trainee rate of different grades. (22)

$\frac{\partial^2 TC}{\partial k_i Q_j}$ =0, where i $\neq$ j , since k_i and Q_j are the trainee rate and batch size of different grades. (23)

$$\frac{\partial^2 TC}{\partial Q_i k_i} = \frac{H_i R_i (1+\alpha_i+\alpha_i^2)}{2k_i^2} > 0 \quad (24)$$

$$\frac{\partial^2 TC}{\partial k_i Q_i} = \frac{H_i R_i (1+\alpha_i+\alpha_i^2)}{2k_i^2} > 0 \quad (25)$$

The Hessian matrix can now be stated as;

$$H_1 = \begin{bmatrix}
\frac{\partial^2 TC}{\partial Q_1^2} & \frac{\partial^2 TC}{\partial Q_1 k_1} & .. & .. & .. \; .. \; .. & .. & .. & .. & .. & & \frac{\partial^2 TC}{\partial Q_1 k_n} \\
\frac{\partial^2 TC}{\partial k_1 Q_1} & \frac{\partial^2 TC}{\partial k_1^2} & \frac{\partial^2 TC}{\partial k_1 Q_2} & .. & .. \; .. & . & .. & & & & \frac{\partial^2 TC}{\partial k_1 k_n} \\
\frac{\partial^2 TC}{\partial Q_2 Q_1} & \frac{\partial^2 TC}{\partial Q_2 k_1} & \frac{\partial^2 TC}{\partial Q_2^2} & \frac{\partial^2 TC}{\partial Q_2 k_2} & .. \; .. \; .. & .. & & & & & \frac{\partial^2 TC}{\partial Q_2 k_n} \\
. & .. & .. & .. & .. \; .. \; .. & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
\cdots \; . & & & & & \frac{\partial^2 TC}{\partial Q_i k_{i-1}} & \frac{\partial^2 TC}{\partial Q_i^2} & \frac{\partial^2 TC}{\partial Q_i k_i} & & & \\
. & & & & & & \frac{\partial^2 TC}{\partial k_i Q_i} & \frac{\partial^2 TC}{\partial k_i^2} & \frac{\partial^2 TC}{\partial k_i Q_{i+1}} & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
\frac{\partial^2 TC}{\partial k_n Q_1} & .. & .. & .. & & & & & .. \; .. & \frac{\partial^2 TC}{\partial k_n Q_n} & \frac{\partial^2 TC}{\partial k_n^2}
\end{bmatrix}$$

International Journal of Science and Technology

http://www.ejournalofsciences.org

After making use of the above equations (19), (21) to (23) we get the matrix in the following form;

$$
H_1 = \begin{bmatrix}
\frac{\partial^2 TC}{\partial Q_1^2} & \frac{\partial^2 TC}{\partial Q_1 k_1} & 0 & 0 & \cdots & & 0 & 0 & \cdot\cdot & 0 & 0 \\
\frac{\partial^2 TC}{\partial k_1 Q_1} & \frac{\partial^2 TC}{\partial k_1^2} & 0 & 0 & \cdot\cdot \; \cdot\cdot \;\; . & & 0 & 0 & & 0 & 0 \\
0 & 0 & \frac{\partial^2 TC}{\partial Q_2^2} & \frac{\partial^2 TC}{\partial Q_2 k_2} & \cdots & & 0 & 0 & \cdot\cdot\;\cdot\cdot\;\cdot\cdot & 0 & 0 \\
. & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot\;\cdot\cdot\;\cdot\cdot & & & & & 0 & . \\
. & & & & & & & & & & \\
. & 0 & & & & & & & & \cdot\cdot & . \\
. & 0 & & & & & & & & & . \\
. & & & & & & & & & & \\
. & & & & & 0 & \frac{\partial^2 TC}{\partial Q_i^2} & \frac{\partial^2 TC}{\partial Q_i k_i} & & & 0 \\
. & & & & & & \frac{\partial^2 TC}{\partial k_i Q_i} & \frac{\partial^2 TC}{\partial k_i^2} & 0 & & 0 \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
0 & & & & & & & & & & 0 \\
0 & 0 & 0 & 0 & & & & & & \frac{\partial^2 TC}{\partial k_n Q_n} & \frac{\partial^2 TC}{\partial k_n^2}
\end{bmatrix}
$$

After applying equations (18), (20), (24) and (25) it is observed that the above matrix is positive definite. Hence the objective function in equation (11) is convex. Hence, it is the global minimum solution.