ISSN 2224-3577
International Journal of Science and Technology

http://www.ejournalofsciences.org

Bifurcation Diagrams of Nonlinear RLC Electrical Circuits

AJIDE, O.O, T.A.O. SALAU
Department of Mechanical Engineering,
University of Ibadan, Nigeria

ABSTRACT

This work investigates the application of bifurcation diagrams in the chaotic study of nonlinear RLC electrical circuits. The relevant second order differential equations were solved for ranges of appropriate parameters using Runge-Kutta approach. The solutions obtained were employed to produce bifurcation diagrams using Microsoft excel 2007. The mean estimate of $\delta = 4.669$ obtained from the bifurcation diagrams is an approximate value of the Feigenbaum constant and thus conforms to the expected results. The results show that bifurcation diagram is a useful tool for exploring dynamics of nonlinear resonant circuit over a range of control parameters.

Keywords: *Bifurcation, RLC electrical circuits, Chaotic systems, Feigenbaum constant*

1. INTRODUCTION

Nonlinear dynamics study of systems governed by equations in which a small change in one variable can induce a large systematic change is known as chaos. Unlike a linear system, in which a small change in one variable produces a small and easily quantifiable systematic change, a nonlinear system exhibits a sensitive dependence on initial conditions. Chaotic behaviour has been observed in nonlinear electrical circuits, oscillating systems, chemical reactions, magneto-mechanical devices, weather and climate etc. Bifurcation diagram can be described as a record of change in the behaviour of a dynamical system as parameter changes. Bifurcation diagram is a useful tool in viewing more globally the dynamics of chaotic system over a range of parameter values, thereby allowing simultaneous comparison of both periodic and chaotic behaviours (Ajide, 2008). Some interesting works have been done using bifurcation diagrams in the study of chaotic systems.

Han *et al* (1995) developed a model and employed bifurcation diagram as a tool for investigating chaotic phenomena in a vibratory ball milling system. The recent developments in understanding the nature of chaos is making it a possibility to tackle it in real-world systems. A framework to model real-world chaotic systems from their short, noisy, observed data, to understand their behavioural changes with respect to time and parametric space has been performed in qualitative analysis by constructing the bifurcation diagrams (Farugi and Kumar, 2005). Joseph (2008) developed a model and bifurcation diagrams of chaotic frequency scaling in a coupled oscillator model for free rhythmic actions. Aisida (2010) investigated the synchronization in a coupled Duffing – Van der pol oscillators with ϕ^6 potential. He concluded that the implementation of an appropriate algorithms and computer source codes generates interesting bifurcation diagrams and poincare map of chaotic dynamic systems. Research shows that a fully synchronization is obtained when K reaches a certain threshold $Kth \simeq 0.2587$. Literatures review shows that extensive work has not been done on the application of bifurcation diagrams in the study of nonlinear RLC circuits' dynamics. This research gap strongly motivates this study.

2. MODEL DESCRIPTIONS

An electrical circuit is a network that has a closed loop, giving a return path for current. An electrical network is an interconnection of electrical elements such as resistors, inductors, capacitors, transmission lines, voltage sources and switches. An RLC circuit (also known as a resonant circuit) is an electrical circuit consisting of a resistor R, an inductor (L), and a capacitor (C) connected in series or parallel. This configuration forms to a harmonic oscillator. Every RLC circuit consists of two components: a power source and resonator. Given the parameters V,R,L and C, the solution for the charge Q, can be found using Kirchhoff's voltage law (KVL).

This gives:

$$V_R + V_L + V_C = V \tag{1}$$

For a time-changing voltage, v (t) becomes

$$Ri(t) + \frac{Ldi}{dt} + \frac{1}{c}\int_{-\infty}^{t} i(t)dt = V(t) \tag{2}$$

Using $i(t) = {dq}/{dt}$

According to Wikipedia (2008), the equation (2) can be expressed in terms of charges across the capacitor as in the equation (3)

$$(Ld^2q/dt^2) + (Rdq/dt) + (1/C) .q (t) \tag{3}$$

A much more elegant way of recovering the circuit properties of RLC circuit is through the use of nondimensionlization. For a parallel configuration of same components, where ϕ is the magnetic flux in the system, the equation (3) becomes:

$$\frac{Cd^2\phi}{dt^2}+\left(\frac{1}{R}\right)\frac{d\phi}{dt}+\left(\frac{1}{L}\right)\phi = i_o Cos(wt) \quad (4)$$

As reported by Gregory and Jerry (1990), an electrical circuit with resistance (R), inductance (L) and nonlinear capacitance (c) may be driven sinusoidally into chaotic states. The differential equation is modeled similar to equation 4 as follows:

$$\frac{d^2x}{dt^2}+\frac{Adx}{dt}+x^3 = B\ Cos(wt) \quad (5)$$

$$\ddot{x}+A\dot{x}+x^3 = B\ Cos(wt) \quad (6)$$

Where $R=\ddot{x}=\frac{d^2x}{dt^2}$, $L=\dot{x}=\frac{Adx}{dt}$ and

A and B are adjustable parameters. It has been suggested that the transition to chaos may be observed for parameter values A = 0.1 and $9.8 < B < 13.4$ (Moon, 1987).

3. NUMERICAL SIMULATIONS

Equation (6) is solved numerically and solutions were obtained using fourth-order Runge-Kutta and FORTRAN 90 sources codes.

4. RESULTS AND DISCUSSIONS

Fig. 1 is the bifurcation diagram of a nonlinear RLC circuit dynamical system when the inductance coefficient A = 0.5, Angular frequency $w=1$, number of slice, N_{slice} = 1000 and at a set of initial conditions $x_o = 2$ and $y_o = 2$

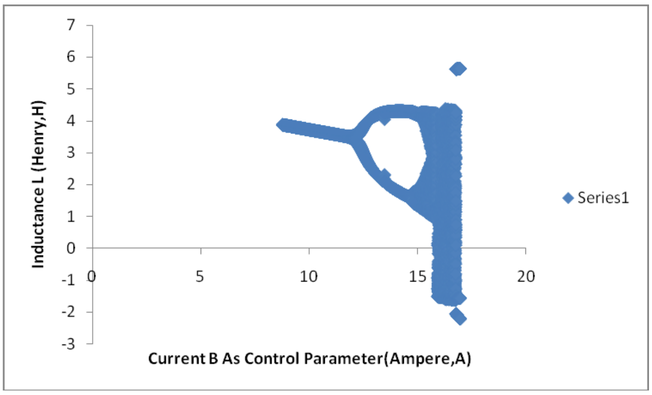

Fig 1: Bifurcation Diagram of Nonlinear RLC Circuits Model (A=0.5, ω=1,N_{slice}=1000)

A stable solution of three thousand cycles at constant time step of 0.01 and tolerance value 0.000001 was achieved after 30 complete cycles are sacrificed (Ns) while 30 cycles are equally examined (Ne). One set of control parameter, which contains chaotic dynamics is $8.80 Amperes < B < 16.79 Amperes$. Between the current B of 8.80 and 11.28Amperes, a near-symmetry is obtained approximately at the inductance of 3.56Henrys. The system bifurcates at 12.01Amperes and 3.40278Henrys.A wide periodic behaviour is experienced in the range $12.01 Amperes < B < 15.43 Amperes$ .Two pairs of period-doubling cascades begins at 14.75Amperes leading to a well pronounced chaotic region in the range $15.43 Amperes < B < 16.79 Amperes$ .The interpretation is that when chaotic behaviour is desirable in the system probably as a measure of control, the current flow should be allowed to be in the range $15.43 Amperes < B < 16.79 Amperes$ Otherwise, the range should be ignored.

An interesting and different dynamics is experienced as shown in Fig. 2 when most of the parameters values are altered. The inductance coefficient changes from A = 0.5 to A = 0.45 with initial conditions $x_o = 3$, $y_o = 3$ and $w_f = 1$. For better accuracy, the number of slice, N_{slice} = 2000 is used instead of 1000.

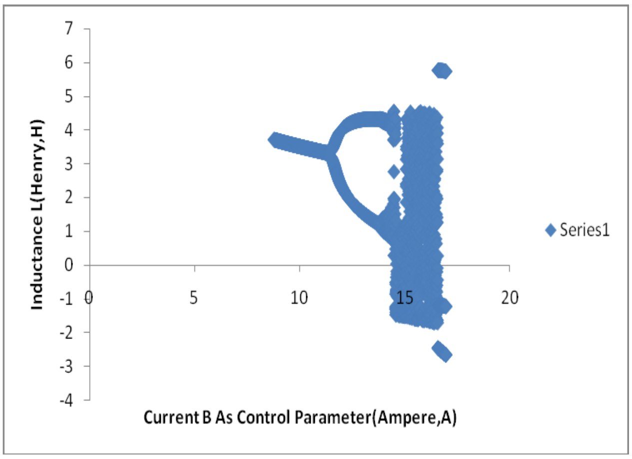

Fig 2: Bifurcation Diagram Of Nonlinear RLC Circuits Model (A=0.45, ω=1, Nslice=2000)

To achieve stable solution of cycles, number of run away cycles, Ns = 30 and those examined, Ne = 30. A near – symmetry is observed at approximately currents of 8.8Amperes with 3.71173Henrys and 10.7Amperes with 3.40741Henrys of inductance. It bifurcates at 11.11Amperes with 3.40741Henrys and leads to the major period window in the range $11.19 Amperes < B < 13.09 Amperes$. This is followed by the two pairs of period-doubling which begins at approximately current of 14Amperes leading to a well defined chaotic region in the range

$14.47 Amperes < B < 16.59 Amperes$. The simple deduction here is that this range of current should be avoided when chaotic behaviour is a disadvantage.

To obtain fig. 3, the inductance coefficient is reduced to A = 0.4 using the set of initial conditions $x_o = 3,\ y_o = 3,\ w = 1$ and N_{slice} = 1000. A period-doubling begins at 10.20Amperes and leads to a periodic window in the range $10.53 Amperes < B < 14.11 Amperes$. Two pairs of period-doubling occur at 14.11Amperes and 13.01Amperes with the inductances of 4.25392Henrys and 0.86199Henry respectively. These pairs of period-doubling leads to chaotic behaviour in the range $14.11 Amperes < B < 16.89 Amperes$. The interpretation of this dynamics is that when chaotic behaviour is advantageous, the range $14.11 Amperes < B < 16.89 Amperes$ should be adopted. Otherwise, the range should be ignored.

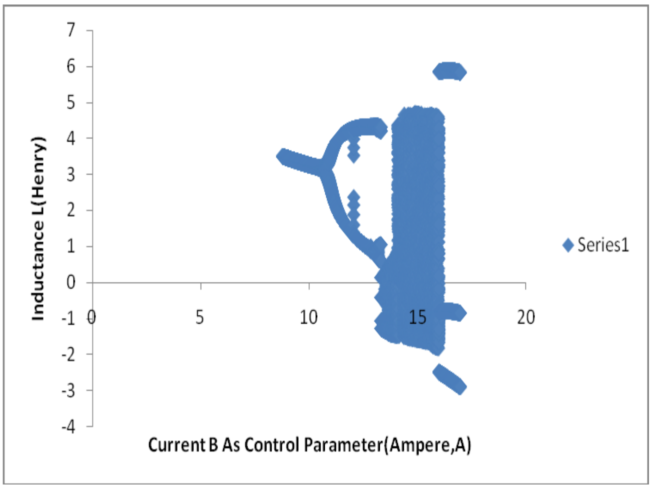

Fig 3: Bifurcation Diagram Of Nonlinear RLC Circuits ($A=0.4, \omega=1, N_{slice}=1000$)

Fig. 4 depicts another very interesting dynamical behaviour of this system using the same parameters as used in figure 3 but using different initial conditions $x_o = 1$ and $y_o = 1$. A pair of period-doubling commences at 10.37Amperes and 3.18452Henrys. A large periodic window is experienced in the range $11.12 Amperes < B < 14.19 Amperes$. Two pairs of period-doubling route to chaos are observed to occur at 13.18Amperes with 4.32968Henrys and 13.02Amperes with 0.88574Henry. A well-defined and wide chaotic region is observed in the range $13.59 Amperes < B < 16.81 Amperes$. When a normal, predictable and non-chaotic situation is desired, the range $11.12 Amperes < B < 14.19 Amperes$ should be employed.

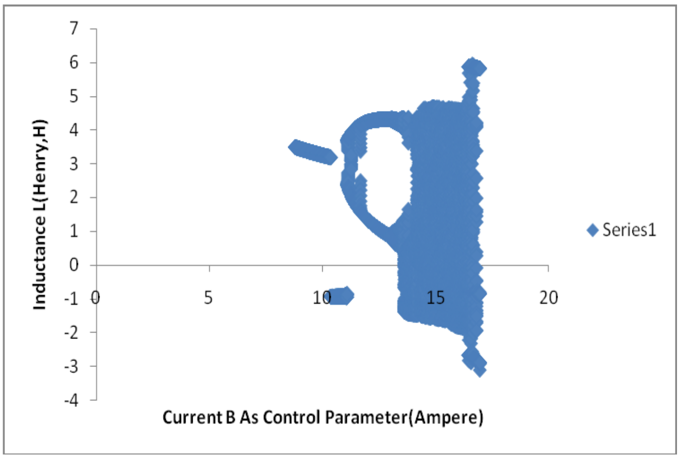

Fig.4: Bifurcation Diagram of Nonlinear RLC Circuits Model ($A=0.4, \omega=1, N_{slice}=1000, x_0=1, y_0=1$)

When the inductance coefficient is reduced to A = 0.35, the number of slice, N_{slice} = 2000 and initial conditions is set to $x_o = 1$ and $y_o = 2$, a different dynamical behaviour is obtained as shown in fig. 5. A near-symmetry is observed from 8.80 to 9.71Amperes. A pair of period-doubling begins at 10.04Amperes and 2.95476Henrys. A well-defined periodic behaviour is experienced in the range $10.28 Amperes < B < 12.88 Amperes$. Two pairs of period-doubling route follow this to chaotic behaviour in the range $12.88 Amperes < B < 16.81 Amperes$. It is also observed that a para-periodic phenomenon is experienced faintly and randomly amidst this chaotic range. What can be deduced from this dynamics is clear. It means that $12.88 Amperes < B < 16.81 Amperes$ is the best range that should be adopted when chaos is necessary to be introduced to the system dynamics perhaps as a measure of control in the RLC circuits.

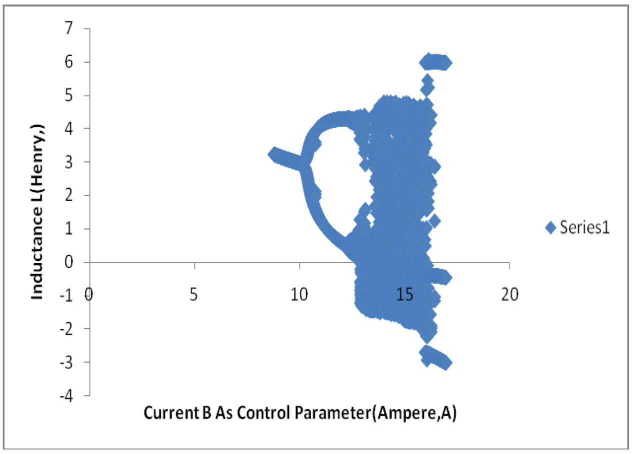

Fig 5: Bifurcation Diagram Of Nonlinear RLC Circuits Model ($A=0.35, \omega=1, N_{slice}=2000, x_0=1, y_0=2$)

As the inductance coefficient is reduced further to A = 0.29 and $w = 1$, N_{slice} = 3000 with the initial conditions set as $x_o = 2$ and $y_o = 2$, the property of chaotic system commonly referred to as sensitivity to initial conditions is again manifested.

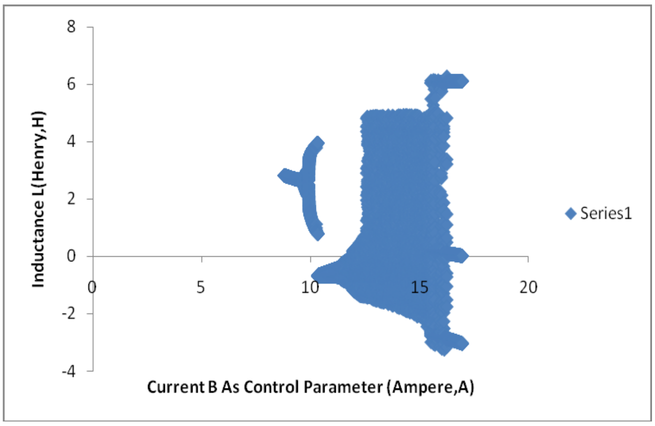

Fig. 6: Bifurcation Diagram of Nonlinear RLC Circuits Model (A=0.29, ω=1, N_{slice}=3000,x_0=2,y_0=2)

As shown in fig. 6, the system bifurcates at 8.88Amperes and 2.79731Henrys. A periodic region in the range $9.62 Amperes < B < 12.58 Amperes$ immediately follows. Two pairs of period-doubling leading to chaos is observed at 11.44Amperes. A very wide chaotic region is experienced in the range $12.58 Amperes < B < 16.82 Amperes$. It can be inferred from this dynamics that the control parameter of between 12.58Amperes and 16.82Amperes should be ignored when chaotic phenomenon is regarded to be problematic.

5. RESULTS VALIDATIONS

The bifurcation diagrams obtained for nonlinear RLC circuits' model are confirmed using Feigenbaum constant (δ) as stated in the equation 7.

$$\delta = \lim_{k-\infty} \frac{\mu_k - \mu_{k-1}}{\mu_{k+1} - \mu_k} \quad (7)$$

$$\delta = \frac{13.7500 - 10.7800}{14.3861 - 13.7500}$$

$$\delta = \frac{2.9700}{0.6261}$$

$$\delta = 4.6691$$

Since $\delta = 4.6691$ estimated from the bifurcation diagram of fig. 2 is approximately equal to the Feigenbaum constant $(\sigma = 4.6692016091029909.....)$ as widely reported and accepted in the literatures, it can be inferred that the bifurcation diagrams validates or satisfy the expected results.

6. CONCLUSIONS

The result of this work reveals that bifurcation diagram is a useful tool for the global view of the dynamics of nonlinear RLC electrical circuits over a range of control parameter. It gives a unique benefit of simultaneous study of both periodic and chaotic behaviour of nonlinear systems.

The work also confirmed that sensitivity to initial conditions is a principal property of all chaotic dynamic systems. A very tiny alteration in the initial conditions gives a distinct bifurcation diagrams and an interesting dynamic behaviours.

REFERENCES

Aisida. O.S. 2010. An unpublished Master of Science (M.Sc) Thesis, Department of Physics, University of Ibadan, Ibadan, Nigeria.

Ajide, O.O.2008. Bifurcation diagrams of some selected chaotic systems. An unpublished Master of Science (M.Sc) submitted to the Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria.

Farugi, M.A. and Kumar, C.S. 2005. Studies on modelling of realworld chaotic systems using computational intelligent approach. A Ph.D Thesis, Department of Mechanical Engineering, IIT, Kharagpur, India.

Gregory, L.B. and Jerry, P.G. 1990. Chaotic dynamics: An introduction. Cambridge University Press, U.S.A.
Han, H.J. Jie, P. and Paul, G.M. 1995. An investigation of chaotic phenomena in a vibratory ball milling systems. Department of Mechanical and Material Engineering, University of Western Australia, Netherlands, WA6009, Australia. Email: hhuang@shiralee.mech.uwe.edu.au.

Joseph, C. 2008. Chaotic frequency scaling in a coupled oscillator model for free rhythmic actions. A paper presentation at the Department of Engineering Science and Mechanics, Pennsylvania State University, A16802, U.S.A jpc3@psu.edu

Moon, F.C. 1987. Chaotic vibrations. 4th Edition, John Wiley and Sons, NY.

Pecora, L.M. and Caroll, T.L. 1990. Synchronization in chaotic systems. Physical review letters 64:821-824.

Wikipedia, F. 2008. RLC circuits – wikipedia, the free encyclopedia. Wikipedia is a registered trademark of the wikipedia foundation Inc., a US Registered 501(c) (3) tax – deductable nonprofit charity. Free encyclopedia.htm