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Investigation of the Solution of a Fredholm Integral Equation of First Kind with un-symmetric Kernel by Using Fourier Series

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ABSTRACT

When an integral equation is solved by using any methods, the results are known as solutions of the integral equation. If the integral equation is solved by using the Fourier series then its solution is called Fourier series solution and this solution represents a stationary signal. Usually, integral equation is solved by the successive approximation method and the resolvent kernel method in which the solution is not of Fourier series type and this solution does not represent a stationary signal. In this paper, our main goal is to determine the solution of a Fredholm integral equation of first kind with unsymmetric kernel by using Fourier series.

Keywords: *Fourier series, Fredholm integral equation, Unsmmetric kernel, stationary signal*

I. INTRODUCTION

Integral equations are one of the most useful techniques in many branches of pure and applied mathematics, particularly in boundary value problems in the ordinary and partial differential equations. Integral equations occur in many fields of mechanics and mathematical physics. They are also related to the problems in mechanical vibrations, theory of analytic functions, orthogonal systems, and quadratic forms of infinitely many variables. Integral equations arise in several problems of science and technology and may be obtained directly from physical problems, i.e. radiation transfer problem and neutron diffusion problem etc. They also arise as representation formula for the solutions of differential equations; a differential equation can be replaced by an integral equation with the help of initial and boundary conditions. As such, each solution of the integral equation automatically satisfies these boundary conditions [1], [5], [6], [9].

The actual development of the theory of integral equations began only at the end of 19^{th} century due to the works of Italian mathematician V. Volterra and principally to the year 1900, in which the Swedish mathematician I. Fredholm published his work on the method of solution for the Direchlet problem [2], [3], [8]. Recently, a new technique has been applied for determining the solution of a Fredholm integral equation of first kind whose name is Fourier series method, where kernel type was symmetric [7].In this paper, we have used this new technique for determining the solution of a Fredholm integral equation of first kind, where kernel type is unsymmetric. Mainly this paper shows that this new technique is not applicable only for symmetric kernel but also applicable for unsymmetric kernel and hence we have just changed kernel in the integral equation of [7] to find our expected result.

II. MATERIALS AND METHOD

In this study, we have used Fourier series method for determining the solution of a Fredholm integral equation of first kind with unsymmetric kernel and also used MATLAB for drawing the graph of the solution.

Integral Equation

An integral equation is an equation in which an unknown function to be determined appears under one or more integral signs.

Examples: 1. $\phi(x)=F(x)+\lambda\int_a^b k(x,t)\phi(t)dt$

2. $\phi(x)=\lambda\int_a^b k(x,t)\phi(t)dt$

The function $\phi(x)$ in (1) and (2) as the unknown function to be determined, while all other functions are known functions. The function $k(x,t)$ is called the kernel of the integral equation and λ is a non-zero real or complex parameter.

If Ω is the domain of the variable t, we often write an integral equation as

$$\phi(x)=F(x)+\lambda\int_\Omega k(x,t)\phi(t)dt$$

Fredholm integral equation

An integral equation is said to be a Fredholm integral equation, if the domain of integration is fixed.

Example: $\alpha(x)\phi(x)=F(x)+\lambda\int_a^b k(x,t)\phi(t)dt$

(1)

i. If $\alpha(x)=0$, then equation (1) reduces to $F(x)=\lambda\int_a^b k(x,t)\phi(t)dt$, this equation is said to be a Fredholm integral equation of first kind. The functions $k(x,t)$ and $F(x)$ and the limits a and b are known. It is proposed to determine the unknown function $\phi(x)$ so that (1) is satisfied for all values of x in the closed interval $a \le x \le b$ and $k(x,t)$ is the kernel of this equation.

ii. If $\alpha(x)=1$, then equation (1) reduces to $\phi(x)=F(x)+\lambda\int_a^b k(x,t)\phi(t)dt$, this equation is said to be a Fredholm integral equation of second kind.

iii. If $F(x)=0, \alpha(x)=1$, then equation (1) reduces to $\phi(x)=\lambda\int_a^b k(x,t)\phi(t)dt$, this equation is said to be a homogeneous Fredholm integral equation of second kind.

III. SYMMETRIC AND UNSYMMETRIC KERNEL

A Kernel $k(x,t)$ is called symmetric (or complex symmetric or Hermitian) if $k(x,t)=\overline{k(x,t)}$. Where the bar denotes the complex conjugate. A real kernel $k(x,t)$ is called symmetric if $k(x,t)=k(t,x)$ and a real kernel $k(x,t)$ is called unsymmetric if $k(x,t)\neq k(t,x)$ [4].

Example: 1. $\sin(x+t), \log(xt)$ are all symmetric kernels

2. $\sin(2x+3t)$ is unsymmetric kernel

Solution of Fredholm integral equation of first kind with unsymmetric kernel

Given an integral equation

$$f(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1-\alpha^2}{1-2\alpha\sin(x-y)+\alpha^2}\phi(y)\,dy$$

$$where, -\pi \le x \le \pi,\ 0<\alpha<1$$

Now, we shall solve this integral equation by using Fourier series.

Solution: The given integral equation is

$$f(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1-\alpha^2}{1-2\alpha\sin(x-y)+\alpha^2}\phi(y)\,dy \quad (2)$$

Which is of the from, $f(x)=\lambda\int_{-\pi}^{\pi}k(x,y)\phi(y)\,dy$ i.e. Fredholm integral equation of first kind.

From equation (2) we obtain

$$k(x,y)=\frac{1-\alpha^2}{1-2\alpha\sin(x-y)+\alpha^2} \text{ and } \lambda=\frac{1}{2\pi}$$

Clearly, the kernel $k(x,y)$ type is unsymmetrical.

Now, $\frac{1-\alpha^2}{1-2\alpha\sin(x-y)+\alpha^2}$

$$=1+2\sum_{n=1}^{\infty}\alpha^n(\sin nx\cos ny-\cos nx\sin ny) \quad (3)$$

The above series is absolutely convergent.

We know that the Fourier series for *f(x)* is defined as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \quad (4)$$

Let, $$\phi(y)=\frac{c_0}{2}+\sum_{n=1}^{\infty}(c_n\cos ny+d_n\sin ny) \quad (5)$$

be the solution of the integral equation (2).

Using the equation (3), (4) and (5) in (2) we obtain

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)=\frac{c_0}{2}+\alpha^n\sum_{n=1}^{\infty}(-d_n\cos nx+c_n\sin nx) \quad (6)$$

Equating the like terms of equation (6) we get

$c_0=a_0,\ b_n=\alpha^n c_n$ and $a_n=-\alpha^n d_n$

$$\Rightarrow c_0=a_0,\ c_n=\alpha^{-n}b_n \text{ and } d_n=-\alpha^{-n}a_n \quad (7)$$

Using the equation (7) in (5) we get

$$\phi(y)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(\alpha^{-n}b_n\cos ny-\alpha^{-n}a_n\sin ny)$$

$$\therefore \phi(y)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\alpha^{-n}(b_n\cos ny-a_n\sin ny) \quad (8)$$

This is the required solution of the given integral equation.

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IV. RESULTS AND DISCUSSION

In this section the solution of the Fredholm integral equation of first kind with unsymmetric kernel are presented graphically. We have drawn Fig. 1, Fig. 2 and Fig. 3 for the solution of the Fredholm integral equation of first kind with unsymmetric kernel by using MATLAB for definite terms of the series and increasing the values of the domain by different step sizes in the interval $[-\pi, \pi]$.

Fig.1 is drawn for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 10$ and step size $y = .01$ in $[-\pi, \pi]$.
Fig. 2 is drawn for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 10$ and step size $y = .5$ in $[-\pi, \pi]$ and
Fig. 3 is drawn for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 20$ and step size $y = .01$ in $[-\pi, \pi]$. We are showing that the following three figures represent a stationary signal (a stationary signal is a signal where there is no change the properties of signal. In other words, a stationary signal is a signal that repeats).
From the Fig. 1 and Fig. 2, we observe that when the step size y is increasing and the values of a_0, α, a_n, b_n and n is keeping fixed, then the wave length is increasing but frequency number is decreasing. Again, from the Fig. 1 and Fig. 3, we observe that when n is increasing and all other values are keeping fixed i.e. the values of a_0, α, a_n, b_n and step size y are unchanged, then the frequency number is increasing but the wave length is decreasing.

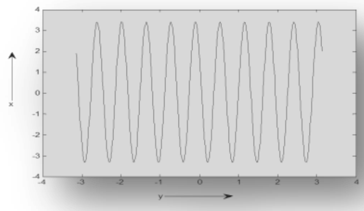

Figure.1. Graph of (8)
for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 10$ and $y = .01$

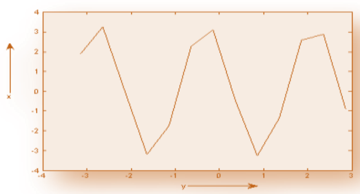

Figure.2. Graph of (8)
for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 10$ and $y = .5$

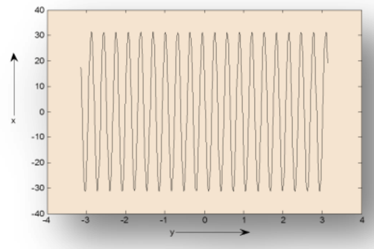

Figure.3. Graph of (8)
for $a_0 = .1, \alpha = .8, a_n = .3, b_n = .2, n = 20$ and $y = .01$

V. CONCLUSION

We know that Fourier series is a powerful tool for analyzing the stationary signal and periodic function is one kind of stationary signal. Also we know that *sine* and *cosine* functions are the examples of the periodic functions. Since *sine and cosine* terms are present in the solution of the Fredholm integral equation of first kind with unsymmetric kernel, hence from our discussion, we can say that the solution of the Fredholm integral equation of first kind with unsymmetric kernel is also a stationary signal. Finally, we conclude that Fourier series is a very interesting, powerful and easy technique for finding the solution of the Fredholm integral equation of first kind with unsymmetric kernel as like as with symmetric kernel.

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