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Mean - Trend Variance Flexibility Portfolio Model

[1]Guochun Wang , [2]Xiaoling Zhou , [3]LiangLiu, [1]XingYu
Department of Mathematics & Applied Mathematics Humanities Science &
Technology Institute of Hunan Loudi, 417000, P.R. China

ABSTRACT

This paper study the basic analysis such as financial analysis, technical analysis - trend analysis, portfolio analysis, We proposed Mean - trend variance flexibility portfolio model, which promote Markowitz mean-variance portfolio model. With time trend to get the risk characteristics of individual securities, while taking advantage of elasticity to the degree associated with a variety of securities analysis, we get the optimal combination of risk to suit the needs of different investors.

Keywords: *basic analysis ; trend analysis; trend variance flexibility portfolio model ; Elasticity*

1. INTRODUCTION

Markowitz [1]used the mean-variance and quadratic programming method to solve the optimal portfolio problem, which is considered as the cornerstone of modern financial theory. Although there are a lot of literature having improved the model, including modifying the model assumptions or the model. For example, some literatures proposed non-normal distribution, Bollerslev (1987)[2] described the foreign exchange return with t-distribution firstly. Hansen (1994)[3] proposed skewed-t-distribution firstly, and considered both capital gains and fat-tail of the skewed nature of consideration. The higher moments are irrelevant to the investor's decision. As a result, in some recent studies, such as [5–9], the concept of Mean–Variance Trade-Off has been extended to include the skewness of return in portfolio selection.

As far as we know there is no literature on the following two issues for discussion. The first is that although in the stock market, there are a lot of objective portfolio investment, but why the investors make portfolio investment, and what portfolio investment on mechanisms and effects are. The second is that stock market investors in addition to reduce portfolio risk, how the optimal choice of investment securities is made according to the relevant information. According to Markowitz's theory, there is no difference when the two securities have the same expected return and variance. In fact, the same expected return and variance of the proceeds of the issue can be different values, so for investors, it is not equivalent to, that is there are differences between them. The first part of this article describes risk characteristics of individual securities with mean - trend variance model. For the second question, Markowitz's model supposed that if the correlation coefficient of securities A and B is ρ, then the correlation coefficient of securities B and A is also ρ, but in practice their interaction in many cases are not equivalent. Therefore, we introduce elasticity to represent the correlation between various types of securities, so that both arrive at the impact of two securities is not equivalent, but also makes the units, compared to more, we give flexibility combined model minimizing risk under the conditions of a given income select and risk in a given context makes gains maximum .

2. THE INVESTMENT PORTFOLIO MODEL

2.1 mean - trend variance model

Suppose there are different securities A, B in the stock market, whose return are $R_{A1}, R_{A2}, R_{B1}, R_{B2}$ with probability $P_{A1}, P_{A2}, P_{B1}, P_{B2}$

Where $\sum_{i=1}^{n} p_i = 1, p_i \geq 0$

We can get that

$$E(R_A) = R_{A1}p_{A1} + R_{A2}p_{A2}$$

$$\sigma_A^2 = p_{A1}E\left(R_{A1} - E(R_{A1})\right)^2 + p_{A2}E\left(R_{A2} - E(R_{A2})\right)^2$$

$$E(R_B) = R_{B1}p_{B1} + R_{B2}p_{B2}$$

$$\sigma_B^2 = p_{B1}E\left(R_{B1} - E(R_{B1})\right)^2 + p_{B2}E\left(R_{B2} - E(R_{B2})\right)^2$$

when $E(R_A) = E(R_B)$; $\sigma_A^2 = \sigma_B^2$,

$$E(R_A) = R_{A1}p_{A1} + R_{A2}p_{A2}$$

In this case, two equations are not the only solution with three unknown available. So R_{A_i} does not necessarily mean R_{B_i} , and it does not necessarily means P_{A_i} as same as P_{B_i} , that is, for investors, they are not equivalent. Similarly, when at time n, the rate of return R_i can not be unique solution. Let us now discuss the solution of the same situation. Suppose two securities gains as follows (return %)

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securities 1	time	2010.1	2010.2	2010.3	2010.4	2010.5	2010.6	2010.7	2010.8
	monthly	10	8	7	6	5	4	3	0

securities 2	time	2010.1	2010.2	2010.3	2010.4	2010.5	2010.6	2010.7	2010.8
	Monthly	0	3	4	5	6	7	8	10

Obviously, the expected rate of return and variance of these two securities are equal. But for rational investors (non-speculators), the securities 2 with smaller risk than securities 1, the situation of securities 1 is getting worse, but securities 2 is contrary.

Suppose there is a securities and it is known of n period earnings (we may assume that the known rate of return per quarter) for ri, the expected return and risk are calculated as follows:

$$E(r)=\frac{1}{n}\sum_{i=1}^{n} r_i\text{ , }\quad \sigma^2(r)=\frac{1}{n}\sum_{i=1}^{n}\left[r_i-E(r)\right]^2$$

We improve this model to mean - trend variance model

$$E(r)=\frac{1}{n}\sum_{i=1}^{n} r_i\text{ ,}$$

$$\sigma'^2(r)=\frac{1}{n}\sum_{i=1}^{n}\left[r_i-E(r)\right]^2\times\log_{n+1}(1+i)$$

where r_i is the actual rate of return, which is in chronological order, $E(r)$ is the expected rate of return, which is the average of each rate of return, $\sigma'^2(r)$ is risk, it shows the size of the risk not only with the degree of deviation from the expected return is also with the overall trend of the yield.

2.2 Flexible combination of model

Securities are not completely independent in stock market. In general, as long as there is between the two economic variables function, we can express the dependent variable with the flexibility to respond to the sensitivity of the independent variables. Therefore, the elasticity can be used to represent the relationship between various types of securities. The general formula of elasticity is:

Elasticity = percentage change in the dependent variable / the proportion of independent variable changes

Now, it is assumed that the first securities as independent variables, the other as the dependent variable. Therefore,

$$E(e_{1i})=\frac{1}{n}\sum_{k=1}^{n}\frac{\Delta X_{ik}}{X_{ik}}/\frac{\Delta X_{1k}}{X_{1k}}$$

$$=\frac{1}{n}\sum_{k=1}^{n}\frac{\Delta X_{ik}}{\Delta X_{1k}}/\frac{X_{ik}}{X_{1k}}$$

where $E(e_{1i})$ is the i-th securities elastic coefficient relating to the first securities, ΔX_{ik} is the changes of the i-th securities in the k-period . X_{ik} is the return of the i-th securities in the k-period.

In this way , we can get association degree of each security with the first one. And

$$E(e_{ji})=\frac{1}{n}\sum_{k=1}^{n}\frac{\Delta X_{ik}}{X_{ik}}/\frac{\Delta X_{jk}}{X_{jk}}$$

That is the elastic coefficient of the i-th securities relating to the j-th securities.

We propose elastic portfolio model maximizing return under the condition of given risk,

$$\max E(r)=\sum_{i=1}^{n} w_i E(r_i)$$

$$s.t\ \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma'^2(r_i)\sigma'^2(r_j)E(e_{ji})\le\sigma^2$$

$$\sum_{i=1}^{n} w_i=1, 0\le w_i\le 1$$

We also propose elastic portfolio model minimizing risk under the condition of given return.

$$\min\sigma'^2=\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma'^2(r_i)\sigma'^2(r_j)E(e_{ji})$$

$$s.t\ \sum_{i=1}^{n} w_i E(r_i)=E(r)$$

$$\sum_{i=1}^{n} w_i=1, 0\le w_i\le 1$$

where $E(r)$ is the return, w_i is the share of the i-th securities, $E(r_i)$)is the expected return of the i-th securities, $E\left(e_{ji}\right)$ is the elastic coefficient of the i-th securities relating to the j-th securities, $\sigma'^{2}\left(r_{i}\right)$ is the trends risk of the i-th securities.

3. EMPIRICAL RESEARCH

We select 14 stocks of the Shenzhen component index, using the above formula and the SPSS software and MALAB, the result is given as the following table:

code	2009.1	2009.2	2009.3	2009.4	2010.1	2010.2	2010.3	2010.4	2011.1	expectation	variance	Trend variance
000001	6.57	6.28	6.21	5.52	7.14	2.83	4.76	4.02	6.68	5.56	1.80	1.46
000002	2.35	4.96	1.15	5.80	2.89	4.18	1.06	8.34	2.65	3.71	4.98	4.10
000024	1.13	2.08	2.93	3.96	2.31	3.80	2.00	2.94	2.65	2.64	0.71	0.42
000027	1.6	7.03	5.05	1.64	3.19	3.37	3.24	0.39	2.00	3.06	3.59	2.22
000039	3.08	3.24	-0.36	0.79	2.44	3.91	8.50	3.65	7.69	3.66	7.35	6.03
000060	0.33	2.50	1.73	5.02	3.35	3.91	1.94	6.02	2.35	3.04	2.89	2.02
000069	2.49	5.41	2.07	6.31	3.07	8.66	2.26	9.01	1.96	4.58	7.24	5.98
000157	5.56	12.4	7.37	6.77	5.49	9.75	5.35	3.51	6.51	6.97	6.27	4.11
000338	4.28	8.92	9.61	6.18	12.1	9.87	7.39	7.27	9.10	8.30	4.69	2.77
000527	3.87	15.9	-2.71	2.96	6.23	9.98	6.49	2.65	4.10	5.50	24.03	13.94
000538	3.49	4.73	3.91	4.73	4.47	6.52	4.79	5.20	4.86	4.74	0.64	0.43
000562	5.69	3.52	5.67	2.73	6.09	5.04	4.69	2.08	4.43	4.44	1.73	1.29
000568	13.25	11.63	7.94	6.57	13.73	10.72	8.07	7.45	13.91	10.36	7.55	5.69
000651	7.02	7.94	7.14	7.12	6.03	8.82	9.29	8.00	6.56	7.55	0.99	0.83

where each column are Code, the first quarter of 2009, and so on until the first quarter of 2011, expectation ,variance, Trend variance.

Elasticity between the various types of securities as

code	000001	000002	000024	000027	000039	000060	000069	000157	000338	000527	000538	000562	000568	000651
000001	1.00	-0.49	-1.31	-3.44	-0.03	-0.88	-0.35	-0.16	1.65	-0.15	-2.72	-0.61	0.51	5.81
000002	-5.17	1.00	4.49	1.98	1.00	1.83	1.20	-7.84	-55.37	-1.42	16.30	-2.15	-14.97	-185.3
000024	-8.15	0.43	1.00	2.62	1.91	0.68	0.29	-0.25	-3.29	-0.10	2.43	-0.05	-2.23	-16.34
000027	-3.82	-0.58	-5.55	1.00	8.95	-1.05	-0.54	1.79	9.10	1.47	-10.30	-0.23	-0.75	30.80
000039	17.55	-0.67	-3.84	-0.99	1.00	-0.64	-0.83	4.01	3.55	0.63	-9.17	-1.56	-3.09	144.9
000060	-19.48	1.22	3.20	1.65	15.19	1.00	1.25	-2.67	-17.68	-0.25	9.14	-2.49	-11.55	-80.64
000069	-1.98	1.26	3.20	5.58	2.34	2.47	1.00	-3.20	-25.89	-0.23	9.95	-1.80	-8.14	-92.49
000157	1.27	0.38	-0.82	3.47	3.28	0.76	0.23	1.00	2.26	0.67	-0.61	0.06	-0.59	4.33
000338	-3.08	-0.20	-0.52	0.61	2.64	-0.53	-0.17	0.08	1.00	0.28	-2.50	0.46	-0.48	15.28
000527	7.54	0.32	-1.61	3.73	8.04	0.38	0.16	3.65	3.94	1.00	-3.40	-0.26	-0.20	96.38
000538	0.45	0.27	0.36	1.91	0.90	0.54	0.19	-0.09	-1.19	0.17	1.00	-0.09	-0.73	-9.29
000562	-3.98	-0.72	-2.02	-0.07	-0.69	-1.12	-0.70	0.11	6.28	0.45	-6.67	1.00	1.87	20.52
000568	4.79	-0.42	-1.60	0.66	-0.14	-0.58	-0.34	-0.18	0.97	0.42	-4.11	0.28	1.00	5.91
000651	0.69	0.22	0.30	0.90	0.43	0.46	0.12	0.23	0.47	0.06	0.71	-0.50	-0.20	1.00

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The correlation coefficient for the various securities

code	000001	000002	000024	000027	000039	000060	000069	000157	000338	000527	000538	000562	000568	000651
000001	1.00													
000002	-0.41	1.00												
000024	-0.53	0.43	1.00											
000027	0.21	-0.34	-0.13	1.00										
000039	-0.16	-0.21	-0.30	-0.18	1.00									
000060	-0.52	0.85	0.71	-0.39	-0.17	1.00								
000069	-0.76	0.86	0.60	-0.21	-0.20	0.81	1.00							
000157	-0.04	-0.03	0.14	0.81	-0.16	-0.18	0.15	1.00						
000338	0.09	-0.16	0.25	0.45	-0.05	0.13	-0.04	0.27	1.00					
000527	-0.14	0.18	-0.15	0.50	0.30	-0.01	0.29	0.69	0.19	1.00				
000538	-0.80	0.42	0.27	-0.03	-0.73	-0.64	0.04	0.35	-0.11	-0.32	1.00			
000562	0.37	-0.84	-0.40	0.27	-0.03	-0.73	-0.64	0.04	0.35	-0.11	-0.35	1.00		
000568	0.51	-0.32	-0.52	0.12	0.26	-0.49	-0.39	0.17	0.29	0.36	-0.15	0.55	1.00	
000651	-0.80	0.09	0.12	0.15	0.40	0.12	0.40	0.21	-0.20	0.36	0.54	-0.27	-0.48	1.00

The coefficient obtained from the above proceeds in the case of flexible combinations of different models of optimal portfolio and optimal Markowitz portfolio model.

The optimal portfolio of flexible combination model							The optimal portfolio of Markowitz model						
Expected return	5	6	7	8	9	10	Expected return	5	6	7	8	9	10
Minimum variance	-74.28	-81.24	-52.90	-28.25	-10.43	1.30	Minimum variance	0.006	0.021	0.0947	0.3705	1.5833	5.4637
000001	0	0	0	0	0	0	000001	0.232	0.314	0.2927	0.0776	0	0
000002	0.6641	0.4036	0.2738	0.2114	0.1491	0.05	000002	0.118	0	0.0294	0	0	0
000024	0	0	0	0	0	0	000024	0.19	0.069	0	0	0	0
000027	0	0	0	0	0	0	000027	0	0	0	0	0	0
000039	0	0	0	0	0	0	000039	0.026	0	0	0	0	0
000060	0	0	0	0	0	0	000060	0	0.087	0	0	0	0
000069	0	0	0	0	0	0	000069	0	0.036	0.0300	0	0	0
000157	0	0	0	0	0	0	000157	0	0	0	0	0	0
000338	0	0	0	0	0	0	000338	0	0	0.0535	0.0778	0.0103	0
000527	0	0	0	0	0	0	000527	0	0	0	0	0	0
000538	0	0	0	0	0	0	000538	0	0	0	0	0	0
000562	0	0	0	0	0	0	000562	0.186	0.061	0	0	0	0
000568	0	0	0.1784	0.449	0.7197	0.94	000568	0	0.037	0.0692	0.1943	0.5133	0.8719
000651	0.3359	0.5964	0.5479	0.3395	0.1312	0	000651	0.242	0.394	0.5252	0.6503	0.4764	0.1281

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Use of the second quarter of 2011 earnings to compare two models

code	00001	00002	00024	00027	00039	000060	000069
The second quarter	5.72	3.79	4.89	2.87	7.78	4.77	5.37
code	000057	000338	000527	000538	000562	000568	000651
The second quarter	6.90	8.30	5.40	6.61	3.82	11.31	8.50
Expected return	5.00	6.00	7.00	8.00	9.00	10.00	
Elasticity return	5.7067	6.7570	7.6377	8.8147	9.9403	10.9500	
M return	5.3721	6.5990	7.7126	8.7651	9.8201	10.9032	

As it can be seen from the above table to get that the elastic model portfolio returns is superior to Markowitz portfolio model.

4. CONCLUSION

This paper obtained a single securities risk-return characteristics with trend variance model, and then use various types of securities introduces cross-correlation between the coefficient of elasticity which will be able to quantify the relationship between various types of securities, and finally we proposed the optimal portfolio model based on flexible combination. In the empirical research, we compare the model result to Markowitz portfolio model, and it shows that our result is better than Markowitz portfolio model by the securities real return of the second quarter of 2011.

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