



Analysis of Convective Plane Stagnation Point Flow with Convective Boundary Conditions

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ABSTRACT

The focus of this study is to carry out a numerical investigation of the convective plane stagnation point flow with convective boundary conditions. The system of coupled partial differential equations was first transformed into a system of coupled ordinary differential equations using the method of similarity transformation, which was then solved using the classical fourth order Runge-Kutta formula together with the shooting techniques. The effects of the thermo-physical parameters were then investigated. The Nusselt number is also determined numerically and compared with the case of flow over a flat plate, It is discovered that the plane stagnation point flow has higher Nusselt numbers due to the presence of the pressure gradient.

Keywords: *plane stagnation point, convective boundary conditions.*

1. INTRODUCTION

Stagnation point flow has become an interesting area of research among scientists and investigators due to its importance in a wide variety of applications both in industrial and scientific applications. Since the work of Crane[1] who studied the two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate(plane stagnation) several other works has been published in literature. For example Ishak et'al[2] studied the stretching sheet due to the presence of magnetic fields considering buoyancy and power law effects. Aman and Ishak[3] also studied the mixed convection flows toward a stagnation point on a stretching sheet using a finite difference scheme.

On the flow with convective boundary conditions, Aziz[4] studied a similarity solution applied to laminar thermal boundary layer flow over a flat plate with a convective surface boundary conditions but only obtained local Biot numbers which he made global on restricted conditions. Makinde and Olanrewaju[5] also presented a study on the buoyancy effects on thermal boundary layer over a flat vertical plate with a convective surface boundary condition, they also obtained local similarity variables which is also global only on certain conditions. Okedayo et'al[6] presented the effects of viscous dissipation on the mixed convection heat transfer over a vertical plate with internal heat generation and convective boundary conditions, their studies also obtained local similarity variables. But the problem of plane stagnation flow over a flat plate with convective boundary condition has been neglected which is the motivation of this present work.

In this paper we present similarity solution to the plane stagnation point flow with convective boundary conditions and obtained global Biot numbers.

2. MATHEMATICAL FORMULATION

A two-dimensional body is placed in a stream of fluid. We consider heat transfer near the upstream stagnation line, where the flow is assumed to be laminar. The problem is restricted to the case of a plane plate perpendicular to the stream. The continuity, momentum and energy equations describing the flow can be written as follows.

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\left. \begin{aligned} u(x,0) = v(x,0) = 0, \\ u(x,\infty) = ax, \\ -k \frac{\partial T}{\partial y}(x,0) = h_f (T_f - T(x,0)), \\ T(x,\infty) = 0 \end{aligned} \right\} \quad (4)$$

Where the u and v are the components of velocity along and normal to the plate respectively. T is the temperature, γ is the kinematic viscosity, α is the thermal diffusivity of the fluid, h_f is the heat transfer coefficient, p is the pressure, ρ is the fluid density, k is the thermal conductivity. In order to transform the above equations (1)-(4) we introduce the following similarity parameters,



$$\left. \begin{aligned} \eta &= y\sqrt{\frac{a}{\gamma}}, \quad \psi(x, y) = x\sqrt{a\gamma}f(\eta), \\ u &= \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \\ pr &= \frac{\gamma}{\alpha}, Bi = h_f\sqrt{\frac{\gamma}{a}} \end{aligned} \right\} \quad (5)$$

With the substitutions of the above variables in equations (1)-(4), we obtain:

$$f''' + ff'' - (f')^2 + 1 = 0 \quad (6)$$

$$\theta'' + pr\theta' = 0 \quad (7)$$

$$\left. \begin{aligned} f'(0) = f(0) = 0, f'(\infty) = 1, \theta'(0) = \\ -Bi(1 - \theta(0)), \theta(\infty) = 0 \end{aligned} \right\} \quad (8)$$

3. NUMERICAL RESULTS AND DISCUSSION

The nonlinear equations (6)-(8) are solved numerically using the classical fourth order Runge-Kutta method with a shooting technique implemented on a computer program written in Maple(14). A convenient step size was chosen to obtain the desired accuracy.

Fig.1 shows the numerical results for a fixed Biot number of 0.05 and for a range of values of the Prandtl number $pr=0.71, 3.0, 7.0$ and 11.4 it is observed that as the Prandtl number increases, the thermal boundary layer thickness decreases also the surface temperature also decreases .

Fig.2 shows the effects of the Biot number $Bi=0.05, 0.2, 0.4$ and 0.8 for constant Prandtl number $pr=0.71$. We observe that increase in the Biot number thickens the thermal boundary layer thickness and also increases the wall temperature.

In table.1 we present the computed values of the Nusselt number, it is observed that the Nusselt number increases with Biot number and the Prandtl number. A numerical comparison is also made with the flat plate, it is observed from the table that the Nusselt number is higher for the plane stagnation point flow due to the presence of the pressure gradient in the plane stagnation point flow(Favre-Marinet and Tardu[7]).

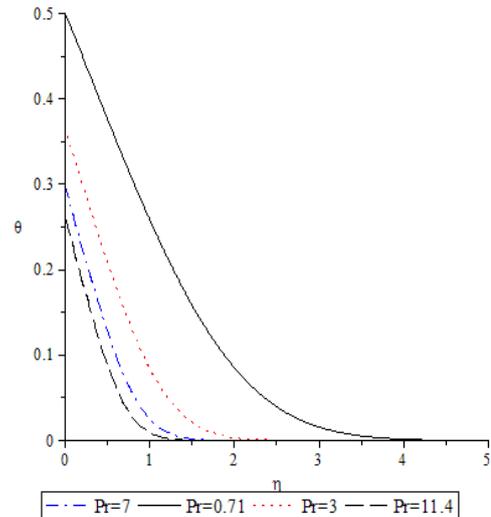


Figure 1: temperature profile for various values of the prandtl number for Bi=0.05

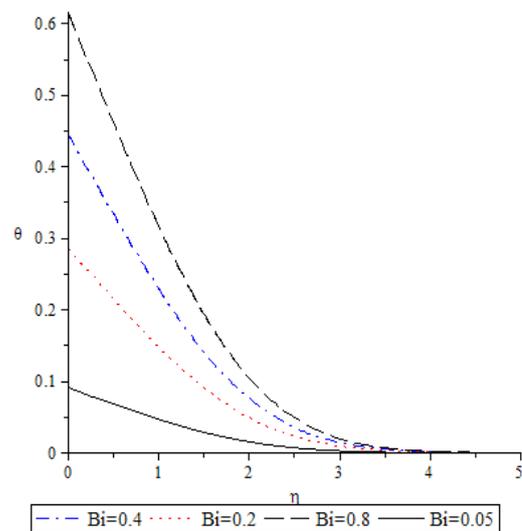


Figure 2: temperature profile for various values of the Biot number for Pr=0.71

Table.1: effects of Biot and Pandtl numbers on the skin friction and Nusselt Numbers.

Bi	Pr	Nusselt Number	Flat plate Nusselt Number
0.05	0.71	0.0454	0.0429
0.2	0.71	0.1428	0.1204
0.4	0.71	0.2220	0.1723
0.8	0.71	0.3072	0.2195
0.05	0.71	0.0454	0.0429
0.05	3.0	0.0473	0.0453
0.05	7.0	0.0480	0.0464
0.05	11.4	0.0483	0.0469



4. CONCLUSION

The problem of plane stagnation point flow has been solved using the method of similarity transform together with the Runge-Kutta method and shooting technique it is observed that the convective boundary conditions and the stagnation point have significant effect on the heat transfer rate and thermal boundary layer thickness.

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