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On the Spectral Expansion Formula for a Class Of Dirac Differential Equations with Piecewise Continuous Coefficient

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ABSTRACT

In this paper, we consider a first order Dirac differential equations system with discontinuous coefficient on the half line $[0,+\infty)$. The resolvent operator is constructed and the expansion formula or equivalently Parseval equation is obtained by applying the method of Titchmarsh.

Keywords: *Dirac operator, expansion formula, resolvent operator.*

1. INTRODUCTION

The expansion formula is obtained by using several methods for regular problems. The most important of these are method of integral equations, the method of contour integration, the finite-difference method, etc. (See [1, 2]). When the coefficient is continuous, the expansion formula is given for the singular problem, considering it as the limit of regular ones in [1, 2]. In this work, the system of Dirac differential equations with piecewise continuous coefficient is discussed and several spectral properties of this system are examined. We use new integral representation obtained for the solution of the system of Dirac equations with discontinuous coefficient in [3]. Applying the method in [4], the resolvent operator is constructed and then the expansion formula for singular problem is obtained by using the contour integration.

We consider the boundary-value problem

$$BY' + \Omega(x)Y = \rho(x)Y, \qquad 0 \le x < \infty \qquad (1)$$

$$y_2(0) = 0, \qquad (2)$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},$$

$$Y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

$p(x)$ and $q(x)$ are real measurable functions, λ is a spectral parameter,

$$\rho(x) = \begin{cases} 1, & x > a, \\ \alpha, & 0 \le x < a \end{cases}$$

and $1 \ne \alpha > 0$.

Assume that the condition

$$\int_0^\infty \|\Omega(x)\| dx < \infty \qquad (3)$$

is satisfied for Euclidean norm of matrix function $\Omega(x)$.

The similar system of equations as in (1) is encountered in non-homogeneous physical events. For example, the applications of discontinuous boundary value problems are given in [5, 6]. In the case of $\rho(x) = 1$, boundary-value problem (1), (2) is examined in [1, 2, 7, 8, 9], also when $\rho(x) \ne 1$, the same problem is investigated with different aspects in [3, 10].

This paper is organized as follows: In Section 1, new integral representation (not operator transformation) for the solution of equation (1) is used and several spectral properties of the problem (1), (2) are examined. In Section 2, the resolvent operator of non-homogenous problem is constructed by assuming that λ is not an eigenvalue of given homogenous boundary-value problem. Finally in Section 3, integrating over expanding paths in the complex λ-plane, we obtain the expansion formula with respect to eigenfunctions.

It is easily shown that the vector function

$$f^0(x,\lambda) = \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{i\lambda\mu(x)}$$

is a solution of equation (1) when $\Omega(x) \equiv 0$, where

$$\mu(x) = \begin{cases} \alpha x - \alpha a + a, & 0 \le x \le a, \\ x, & x > a. \end{cases}$$

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It is known from [3] (see [10]), when the condition (3) is satisfied, the equation (1) has a solution $f(x,\lambda)$, for $\operatorname{Im}\lambda \geq 0$ which satisfies the condition

$$\lim_{x\to\infty} f(x,\lambda)e^{-i\lambda x} = \begin{pmatrix} 1 \\ -i \end{pmatrix} \qquad (4)$$

and is expressed uniquely as

$$f(x,\lambda) = f^0(x,\lambda) + \int_{\mu(x)}^{\infty} K(x,t)\begin{pmatrix} 1 \\ -i \end{pmatrix} e^{i\lambda t}dt. \qquad (5)$$

Moreover, the elements of the matrix kernel $K(x,.)$ are summable on the positive half line and for the Euclidean norm of $K(x,t)$, the inequality

$$\int_{\mu(x)}^{\infty} \|K(x,t)\| dt \leq e^{\sigma(x)} - 1$$

is satisfied, where

$$\sigma(x) = \int_x^{\infty} \|\Omega(t)\| dt .$$

Let $Y(x,\lambda)$ and $Z(x,\lambda)$ be vector solutions of the equations system (1). The expression

$$W[Y(x,\lambda), Z(x,\lambda)] = Y^T(x,\lambda) BZ(x,\lambda) = y_1 z_2 - y_2 z_1$$

is called Wronskian of the vector functions $Y(x,\lambda)$ and $Z(x,\lambda)$. Since the Wronskian of the vector functions $f(x,\lambda)$ and $\overline{f(x,\lambda)}$ which are solutions of the equation (1) doesn't depend on x and

$$W\left[f(x,\lambda), \overline{f(x,\lambda)} \right] = 2i,$$

the vector functions $f(x,\lambda)$ and $\overline{f(x,\lambda)}$ are fundamental solutions system of the equation (1) for real λ.

We denote by $\varphi(x,\lambda)$ the solution of the equation (1) satisfying the conditions

$$\varphi_1(0,\lambda) = -1, \quad \varphi_2(0,\lambda) = 0 .$$

Proposition 1.1. *For real* λ, *the identity*

$$2i\frac{\varphi(x,\lambda)}{f_2(0,\lambda)} = \overline{f(x,\lambda)} - S(\lambda) f(x,\lambda) \qquad (6)$$

is valid, where

$$S(\lambda) = \frac{\overline{f_2(0,\lambda)}}{f_2(0,\lambda)} \qquad (7)$$

and

$$S(\lambda) = \left[\overline{S(\lambda)} \right]^{-1}, \quad |S(\lambda)| = 1.$$

Proof. Since $f(x,\lambda)$ and $\overline{f(x,\lambda)}$ are fundamental solutions system of the equation (1) for real λ,

$$\varphi(x,\lambda) = c_1 f(x,\lambda) + c_2 \overline{f(x,\lambda)}$$

can be written. Taking into account the following relations

$$W[f(x,\lambda), \varphi(x,\lambda)] = f_1(0,\lambda)\varphi_2(0,\lambda) - f_2(0,\lambda)\varphi_1(0,\lambda) = f_2(0,\lambda)$$

and

$$W[\overline{f(x,\lambda)}, \varphi(x,\lambda)] = \overline{f_1(0,\lambda)}\varphi_2(0,\lambda) - \overline{f_2(0,\lambda)}\varphi_1(0,\lambda) = \overline{f_2(0,\lambda)},$$

$$c_1 = -\frac{\overline{f_2(0,\lambda)}}{2i}, \quad c_2 = \frac{f_2(0,\lambda)}{2i}$$

are obtained. Hence

$$\varphi(x,\lambda) = -\frac{\overline{f_2(0,\lambda)}}{2i} f(x,\lambda) + \frac{f_2(0,\lambda)}{2i}\overline{f(x,\lambda)}.$$

We show that for all real λ, $f_2(0,\lambda) \neq 0$. On the contrary, there exist a real number λ_0 such that $f_2(0,\lambda_0) = 0$. Then,

$$f_1(0,\lambda_0)\overline{f_2(0,\lambda_0)} - f_2(0,\lambda_0)\overline{f_1(0,\lambda_0)} = 2i$$

is valid according to expression of Wronskian. The contradiction is obtained, so the assumption is not true. Dividing both sides of the last expression by $f_2(0,\lambda)$, the identity (6) is obtained.

$$S(\lambda) = \left[\overline{S(\lambda)} \right]^{-1}, \quad |S(\lambda)| = 1$$

are found directly from (7). The proposition is proved.

The function $S(\lambda)$ defined by (7) is called scattering function of the boundary value problem (1), (2).

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Proposition 1.2. *The function* $f_2(0,\lambda)$ *has no zeros on the closed upper plane.*

Proof. It is clear from (5) that the function $f_2(0,\lambda)$ can be continued as analytical and is continuous on the whole line. For real λ, $f_2(0,\lambda)\neq 0$ is proved in previous proposition. Let us show that the function $f_2(0,\lambda)$ has no zeros in the half plane ($\operatorname{Im}\lambda>0$). Assume the contrary. Let $\lambda=\mu$ ($\operatorname{Im}\mu>0$) be a zero of the function $f_2(0,\lambda)$. Consider the following equations

$$Bf'(x,\mu)+\Omega(x)f(x,\mu)=\rho(x)\mu f(x,\mu),$$

$$-f^{*'}(x,\mu)B+f^{*}(x,\mu)\Omega(x)=\rho(x)\overline{\mu}f^{*}(x,\mu),$$

where $f^{*}(x,\mu)$ is the transpose of $\overline{f(x,\mu)}$. Multiplying the first equation by $f^{*}(x,\mu)$ and the second equation by $f(x,\mu)$, substracting the first equality from the second one and finally integrating this relation from 0 to ∞, we get

$$W\left[\overline{f(x,\mu)},f(x,\mu)\right]_{x=0}+(\overline{\mu}-\mu)\int_0^{\infty}f^{*}(x,\mu)f(x,\mu)\rho(x)dx=0.$$ Thus, $\mu=\overline{\mu}$ is obtained. It is contrary to assumption. This contradiction shows that $f_2(0,\lambda)$ has no zeros in the half plane ($\operatorname{Im}\lambda>0$).

2. RESOLVENT OPERATOR

In the Hilbert Space $H_\rho=L_{2,\rho}(0,\infty;\mathbb{C}^2)$ an inner product is defined by

$$(F,G)=\int_0^{\infty}\left\{F_1(x)\overline{G_1(x)}+F_2(x)\overline{G_2(x)}\right\}\rho(x)dx$$

for the component vectors

$$F:=\begin{pmatrix}F_1(x)\\F_2(x)\end{pmatrix}\in H_\rho,\quad G:=\begin{pmatrix}G_1(x)\\G_2(x)\end{pmatrix}\in H_\rho.$$

Let us define

$$D(L):=\left\{\begin{matrix}F\,|\,F=(F_1(x),F_2(x))\in H_\rho,F_1(x),F_2(x)\in AC[0,b],\\ [0,b]\subset[0,\infty),\, l(F)\in H_\rho, F_2(0)=0\end{matrix}\right\}$$

as $L:F\to l(F)$, where

$$l(F):=\frac{1}{\rho(x)}\left\{BF'+\Omega(x)F\right\}.$$

The boundary value problem (1), (2) is equivalent to the equation $LY=\lambda Y$ and the operator L with domain $D(L)$ is self-adjoint in the space $H_\rho=L_{2,\rho}(0,\infty;\mathbb{C}^2)$.

If λ is not a spectrum point of operator L, then the resolvent $R_\lambda=(L-\lambda I)^{-1}$ exists. Now we find this expression of the operator R_λ.

Lemma 2.1. *The resolvent* R_λ *is the integral operator with the kernel which has the following form*

$$R_\lambda(x,t)=-\frac{1}{f_2(0,\lambda)}\begin{cases}\varphi(x,\lambda)f^T(t,\lambda), & x\le t,\\ f(x,\lambda)\varphi^T(t,\lambda), & x\ge t,\end{cases}\qquad(8)$$

where $f^T(t,\lambda)$ *denotes the transposed vector function of* $f(t,\lambda)$.

Proof. Let $F(x)\in D(L)$ and $F(x)=\begin{pmatrix}F_1(x)\\F_2(x)\end{pmatrix}$ be zero in exterior of every finite interval $[0,b]\subset[0,\infty)$. To construct the resolvent operator of L, we need to solve the initial value problem

$$BY'+\Omega(x)Y=\lambda\rho(x)Y+\rho(x)F(x),\qquad(9)$$

$$y_2(0)=0.\qquad(10)$$

By applying the method of variation of parameters, we want to find the solution of problem (9), (10) which has a form

$$Y(x,\lambda)=c_1(x,\lambda)\varphi(x,\lambda)+c_2(x,\lambda)f(x,\lambda),\qquad(11)$$

where $f(x,\lambda)$ and $\varphi(x,\lambda)$ are solutions of homogeneous problem. Then we get the equations system

$$\begin{aligned}c_1'(x,\lambda)f^T(x,\lambda)B\varphi(x,\lambda)&=f^T(x,\lambda)F(x)\rho(x),\\ c_2'(x,\lambda)\varphi^T(x,\lambda)Bf(x,\lambda)&=\varphi^T(x,\lambda)F(x)\rho(x).\end{aligned}\qquad(12)$$

Since $Y(x,\lambda)\in L_{2,\rho}(0,\infty;\mathbb{C}^2)$, then $c_1(\infty,\lambda)=0$. Using this relation system and (12), we get

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$$c_1(x,\lambda) = -\frac{1}{f_2(0,\lambda)}\int_x^\infty f^T(t,\lambda)F(t)\rho(t)\,dt\,, \quad (13)$$

$$c_2(x,\lambda) = c_2(0,\lambda) - \frac{1}{f_2(0,\lambda)}\int_0^x \varphi^T(t,\lambda)F(t)\rho(t)\,dt\,. \quad (14)$$

Substituting (13) and (14) into (11), we obtain

$$Y(x,\lambda) = \int_0^\infty R_\lambda(x,t)F(t)\rho(t)\,dt + c_2(0,\lambda)\,f(x,\lambda),$$

where

$$R_\lambda(x,t) = -\frac{1}{f_2(0,\lambda)}\begin{cases}\varphi(x,\lambda)\,f^T(t,\lambda), & x \le t,\\ f(x,\lambda)\varphi^T(t,\lambda), & x \ge t.\end{cases}$$

Taking the condition (10), we get $c_2(0,\lambda) = 0$. Thus

$$(L-\lambda I)^{-1}F = \int_0^\infty R_\lambda(x,t)F(t)\rho(t)\,dt\,, \quad F \in D(L)\,. \quad (15)$$

Lemma 2.2. *Let the vector function $F(x)$ be finite at infinity and $F(x) \in D(L)$. Then*

$$\int_0^\infty R_\lambda(x,t)F(t)\rho(t)\,dt = -\frac{F(x)}{\lambda} + \frac{Z(x,\lambda)}{\lambda}\,, \quad (16)$$

where

$$Z(x,\lambda) = \int_0^\infty R_\lambda(x,t)\left[BF'(t) + \Omega(t)F(t)\right]dt\,.$$

Proof. Using the representation of (8),

$$\int_0^\infty R_\lambda(x,t)F(t)\rho(t)\,dt =$$

$$= -\frac{1}{f_2(0,\lambda)}\int_0^x f(x,\lambda)\varphi^T(t,\lambda)F(t)\rho(t)\,dt$$

$$-\frac{1}{f_2(0,\lambda)}\int_x^\infty \varphi(x,\lambda)f^T(t,\lambda)F(t)\rho(t)\,dt$$

$$= -\frac{f(x,\lambda)}{\lambda f_2(0,\lambda)}\int_0^x\left\{-\frac{\partial}{\partial t}\varphi^T(t,\lambda)B + \varphi^T(t,\lambda)\Omega(t)\right\}F(t)\,dt$$

$$-\frac{\varphi(x,\lambda)}{\lambda f_2(0,\lambda)}\int_x^\infty\left\{-\frac{\partial}{\partial t}f^T(t,\lambda)B + f^T(t,\lambda)\Omega(t)\right\}F(t)\,dt$$

can be written. Integrating by parts the last equation, we obtain (16).

The expression

$$\lim_{\substack{\lambda\to\infty\\ \lambda\in\Gamma_R}}\ \sup_{x\ge 0}\left|Z(x,\lambda)\right| = 0 \quad (17)$$

is hold, where Γ_R is a circle having radius R and center at zero. In fact, for $\lambda \to \infty$,

$$\varphi_k(x,\lambda) = O\left(e^{\operatorname{Im}\lambda(\mu(x)-(-\alpha a+a))}\right),$$

$$f_k(x,\lambda) = O\left(e^{\operatorname{Im}\lambda\mu(x)}\right), \qquad k = 1,2 \quad (18)$$

and for $\operatorname{Im}\lambda \ge 0$

$$\left|f_2(0,\lambda)\right| \ge e^{-\operatorname{Im}\lambda(-\alpha a+a)} \quad (19)$$

are valid. Using (18) and (19), we obtain (17).

The following lemma is well known from [8].

Lemma 2.3. $\overline{R_\lambda} = R_{\bar\lambda}$.

3. EXPANSION FORMULA

Theorem 3.1. *The expansion formula which is equivalent to Parseval equality*

$$\delta(t-x) = \frac{1}{4\pi}\int_{-\infty}^{\infty} u(x,\lambda)u^*(t,\lambda)\rho(t)\,d\lambda \quad (20)$$

is hold, where δ is Dirac delta function, as $x \to \infty$

$$u(x,\lambda) = e^{-i\lambda x}\begin{pmatrix}1\\ i\end{pmatrix} - S(\lambda)e^{i\lambda x}\begin{pmatrix}1\\ -i\end{pmatrix} + o(1)\,, \quad (21)$$

$$u^*(t,\lambda) = e^{i\lambda t}\begin{pmatrix}1\\ -i\end{pmatrix}^T - \overline{S(\lambda)}e^{-i\lambda t}\begin{pmatrix}1\\ i\end{pmatrix}^T + o(1)\,, \quad (22)$$

$u^*(t,\lambda)$ is the transpose of $\overline{u(t,\lambda)}$.

Proof. With the help of the Lemma 2.1, Lemma 2.2 and Lemma 2.3, we obtain the expansion formula. We integrate

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both sides of (16) with respect to λ over Γ_R. As a result, we have

$$-F(x)=\frac{1}{2\pi i}\oint_{\Gamma_R} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt+\varepsilon_R(x), \quad (23)$$

where

$$\varepsilon_R(x)=-\frac{1}{2\pi i}\oint_{\Gamma_R}\frac{Z(x,\lambda)}{\lambda}d\lambda .$$

The function $R_\lambda(x,t)$ is analytic in the upper and lower half plane. Therefore we have

$$\frac{1}{2\pi i}\oint_{\Gamma_R} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt=I_R^1+I_R^2+I_R^3, \quad (24)$$

where

$$I_R^1=\frac{1}{2\pi i}\int_{-R-i\varepsilon}^{R-i\varepsilon} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt, \quad (25)$$

$$I_R^2=\frac{1}{2\pi i}\int_{R+i\varepsilon}^{-R+i\varepsilon} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt, \quad (26)$$

$$I_R^3=\frac{1}{2\pi i}\left\{\int_{R-i\varepsilon}^{R+i\varepsilon} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt+ +\int_{-R+i\varepsilon}^{-R-i\varepsilon} d\lambda\int_0^\infty R_\lambda(x,t)F(t)\rho(t)dt\right\}$$

and here ε is any positive number. Using Lemma 2.2 and (17), as $R\to\infty$, $I_R^3\to 0$ and $\lim_{R\to\infty}\sup_{x\ge 0}|\varepsilon_R(x)|=0$ is obtained. From (8), we find

$$\lim_{\varepsilon\to 0} R_{\lambda\pm i\varepsilon}=R_{\lambda\pm i0}.$$

Thus,

$$\lim_{\varepsilon\to 0}\int_0^\infty R_{\lambda\pm i\varepsilon}(x,t)F(t)\rho(t)dt=\int_0^\infty R_{\lambda\pm i0}(x,t)F(t)\rho(t)dt .$$

Therefore, by going over in (23) to the limit as $R\to\infty$ and using (24)-(26), we get

$$F(x)=-\frac{1}{2\pi i}\int_{-\infty}^\infty d\lambda\int_0^\infty\left[R_{\lambda-i0}(x,t)-R_{\lambda+i0}(x,t)\right]F(t)\rho(t)dt. \quad (27)$$

Let $\psi(x,\lambda)$ be the solution of equation (1) with the initial condition

$$\psi(0,\lambda)=\begin{pmatrix}0\\1\end{pmatrix}.$$

Then

$$f(x,\lambda)=f_2(0,\lambda)\psi(x,\lambda)-f_1(0,\lambda)\varphi(x,\lambda). \quad (28)$$

Since $R_{\lambda-i0}=\overline{R}_{\lambda+i0}$ from Lemma 2.3, in addition to $\varphi(x,\lambda)$ and $\psi(x,\lambda)$ are entire functions of λ, it follows from (8) and (15) that,

$$-\int_0^\infty\left[R_{\lambda+i0}(x,t)-\overline{R}_{\lambda+i0}(x,t)\right]F(t)\rho(t)dt=$$

$$=\int_0^x\left[\frac{f(x,\lambda)}{f_2(0,\lambda)}-\overline{\frac{f(x,\lambda)}{f_2(0,\lambda)}}\right]\varphi^T(t,\lambda)F(t)\rho(t)dt$$

$$+\int_x^\infty\varphi(x,\lambda)\left[\frac{f^T(t,\lambda)}{f_2(0,\lambda)}-\overline{\frac{f^T(t,\lambda)}{f_2(0,\lambda)}}\right]F(t)\rho(t)dt.$$

On the other hand, using (28) and the expression of Wronskian

$$\frac{f(x,\lambda)}{f_2(0,\lambda)}-\overline{\frac{f(x,\lambda)}{f_2(0,\lambda)}}=\left[\overline{\frac{f_1(0,\lambda)}{f_2(0,\lambda)}}-\frac{f_1(0,\lambda)}{f_2(0,\lambda)}\right]\varphi(x,\lambda)$$
$$=-2i\frac{\varphi(x,\lambda)}{|f_2(0,\lambda)|^2}$$

is found. Therefore,

$$\int_0^\infty\left[R_{\lambda+i0}(x,t)-R_{\lambda-i0}(x,t)\right]F(t)\rho(t)dt=$$

$$=\frac{1}{\pi}\int_0^\infty\frac{\varphi(x,\lambda)\varphi^T(t,\lambda)}{|f_2(0,\lambda)|^2}F(t)\rho(t)dt.$$

We obtain the following expansion with respect to eigenfunctions of the operator L on the form

$$F(x)=\frac{1}{\pi}\int_{-\infty}^\infty d\lambda\int_0^\infty\frac{\varphi(x,\lambda)\varphi^T(t,\lambda)}{|f_2(0,\lambda)|^2}F(t)\rho(t)dt \quad (29)$$

or

$$F(x)=\frac{1}{4\pi}\int_{-\infty}^\infty d\lambda\int_0^\infty u(x,\lambda)u^*(t,\lambda)F(t)\rho(t)dt. \quad (30)$$

The expansion formula or equivalently Parseval equality (20) is derived from (29). Furthermore, (21) and (22) asymptotic

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formula are found directly from the condition (4). The theorem is proved.

By writing the equation (29) in the form of Stieltjes integral, we have

$$F(x)=\int_{-\infty}^{\infty}\varphi(x,\lambda)\left(\int_{0}^{\infty}\varphi^{T}(t,\lambda)F(t)\rho(t)\,dt\right)d\sigma(\lambda),$$

where $\sigma(\lambda)$ is called spectral function of L operator and has the following form

$$d\sigma(\lambda)=\frac{1}{\pi}\frac{d\lambda}{|f_2(0,\lambda)|^2},\qquad -\infty<\lambda<\infty.$$

Now, by taking the following notation

$$\Phi(\lambda)=\int_{0}^{\infty}\varphi^{T}(x,\lambda)F(x)\rho(x)\,dx,$$

it follows from (29) that

$$F(x)=\int_{-\infty}^{\infty}\Phi(\lambda)\varphi(x,\lambda)\,d\sigma(\lambda).$$

Multplying both sides of this equation by $F(x)$ and integrating it, we obtain Parseval equality as follows

$$\int_{0}^{\infty}F^2(x)\rho(x)\,dx=\int_{-\infty}^{\infty}\Phi^2(\lambda)\,d\sigma(\lambda).$$

4. CONCLUSIONS

When the condition (3) is satisfied, (1), (2) boundary value problem has not discrete spectrum. The spectrum of the problem is continuous and fills the whole line $(-\infty,\infty)$.

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