



Fitting and Benchmarking of Monte Carlo Output Parameters for ^{192}Ir High Dose Rate Brachytherapy “New or v2” Source

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ABSTRACT

A robust model to directly represent output from contemporary brachytherapy calculation had been proposed and developed to make clinical implementation practicable. The model was used to minimize inherent uncertainties associated with the AAMP TG-43 formalism, with consolidation of the TG-43 parameters into new parameter to speed up computational time. Analytical computations were performed to hybridize the anisotropy function, $F(r, \theta)$ and radial dose function, $g(r)$ into a single new function $f(r, \theta)$ for a Nucletron microSelectron High Dose Rate “new or v2” (mHDRv2) ^{192}Ir brachytherapy source. Outputs obtained with the new function were curve fitted with MatLab and compared to those of Monte Carlo serving as the standard to evaluate the average and maximum uncertainties. Dose rate deviations between the Monte Carlo data and MatLab fits, quantified as errors showed minimal deviations less than 0.3%. In comparison with Lliso’s study, this work shows marginal improvement in the deviated values. This work has shown much promise in building robust models to represent outputs from contemporary brachytherapy dosimetry calculations.

Keywords: *Brachytherapy, dosimetry, functional fit, hybridize new function, MatLab application software, treatment planning systems.*

1. INTRODUCTION

Brachytherapy, the use of radioactive sources for the treatment of tumors is an important tool in radiation oncology with the Medical Physics staff responsible for calculating and delivering accurate doses. Treatment Planning Systems (TPS) have now become a standard practice for dose computations in many Radiation Oncology Departments. Independent calculations to verify these results by the commercial TPS are an important part of a good quality management system for brachytherapy implants. Inherent errors in dose distributions are produced by these TPSs due to its failure to account for heterogeneity in the calculation algorithms and the Monte Carlo (MC) method seems to be the panacea for these corrections. Over the years, research works have been performed to revise dose calculation formalisms and to advocate the revision of basic dosimetry data for various brachytherapy sources. Many TPSs rely on functional fits of radial dose function and anisotropy function data when calculating the dose distribution around a brachytherapy seed. This smoothes the original data to allow the calculation of lookup tables and furthermore, could be used directly in dosimetry calculations. The fifth order polynomial data fits were offered in the TG-43 publication, Furhang and Anderson used the double exponential, Moss used the modified sigmoidal, and Meigooni et al used the modified fifth order polynomial to fit for $g(r)$ [1, 2, 3]. Furhang and Anderson again came out with a fit for anisotropy function for ^{192}Ir and ^{125}I [1].

Lliso et al published a functional fitting form for a high dose rate ^{192}Ir brachytherapy sources currently in use [4]. These fits are very close to Monte Carlo output parameters [5]. The proposed functional forms by Lliso et al can reproduce $F(r, \theta)$ and functions for nonsymmetrical sources from $\theta = 0$ to $\theta = \pi$ giving accurate values of these functions in a simple form suitable for use in treatment planning systems, an advantage over the other forms of functional fits [6, 7]. Published data of anisotropy function, $F(r, \theta)$ and Radial dose function, $g(r)$ fitted with functional forms utilized 32 parameters and 9 different formulae to compute dose rate distribution. Although a high degree of accuracy was achieved by this method, a lot of task is placed on the computer memory, storage capacity and speed.

2. MATERIALS AND METHODS

MatLab application software ‘MATLAB 7.0.1’ version (service pack 1) produced by the MathWorks Inc [8] was used for the curve fitting of the various parameters and the fit results copied to tables in Microsoft Excel 2007 spreadsheet for graphical and numerical comparison.

There is a wealth of literature on functional fittings of both radial dose functions and anisotropy functions for various brachytherapy sources and the method employed by Lliso et al has been the most versatile, applying accurately even to non-symmetrical sources [4, 6]. Therefore, the formulae by Lliso et al was



adopted as the standard for calculating the dose rate distribution around the high intensity ¹⁹²Ir mHDRv2 design source [9] in this study. Characteristics of the ¹⁹²Ir source employed in the study are given in Table 1.

2.1 Curve Fitting

The dose calculation formalism was based on the modification of TG-43 parameters to represent the outputs from direct MC [10]. The TG-43 equation was given by

$$\dot{D}(r, \theta) = S_k \Lambda \left[\frac{G(r, \theta)}{G(r_0, \theta_0)} \right] g(r) F(r, \theta) \tag{1}$$

Where S_k is the source air-kerma strength defined as the product of air kerma rate at a calibration distance, d , in free space, $\dot{K}(d)$, measured along the transverse bisector of the source and the square of the distance. Λ is the dose-rate constant which is defined as the dose rate to water at a distance of 1 cm on the transverse axis of a unit kerma strength source in a water phantom. $G(r, \theta)$ is the geometry factor that accounts for the geometric falloff of the photon fluence with distance from the source and depends on the distribution of radioactive material [11].

$g(r)$ is the radial dose function that accounts for radial dependence of photon absorption and scattering in the medium along the transverse axis ($\theta_0 = \pi/2$).

$F(r, \theta)$ is the anisotropy function that accounts for the effect of angular dependence (anisotropy of dose distribution around the source) of photon absorption and scattering in the medium.

The Monte Carlo output could be expressed as in equation (2) [12].

$$\dot{D}(r, \theta) = K_m^{a,ref} \frac{\left[\frac{D}{R} \right]_c^{W,POS}}{\left[\frac{K}{R} \right]_c^{a,ref}} \tag{2}$$

Where $\left[\frac{D}{R} \right]_c^{W,POS}$ denotes the calculated dose per radiant energy to a point at the position pos in a large water phantom. The calculated kerma per radiant energy in air at the reference point, $\left[\frac{K}{R} \right]_c^{a,ref}$, is used together with the measured air kerma rate in air, $K_m^{a,ref}$ at the same reference point, to determine the absolute dose rate [13].

Equation (2) can be rewritten as

$$\dot{D}(r, \theta) = \dot{K}_m^{a,ref} Cr^2_{100} \frac{\left[\frac{D}{R} \right]_c^{W,(ro,fb)} \left[\frac{D}{R} \right]_c^{W,pos}}{\left[\frac{K}{R} \right]_c^{a,ref} Cr^2 \left[\frac{D}{R} \right]_c^{W,(ro,fb)}} \tag{3}$$

Where

C is the correction factor for converting the reference air kerma strength to kerma in free space at radial distance ($r_{100} = 100cm$).

Comparing equations (1) and (3)

$$S_k = \dot{K}_m^{a,ref} Cr^2_{100} \tag{4}$$

From the TG-43 formalism;

$$\Lambda = \frac{\dot{D}(r_0, \theta_0)}{S_k} = \frac{\dot{D}(r_0, \theta_0)}{\dot{K}_m^{a,ref} Cr^2_{100}} \tag{5}$$

$$\Lambda \equiv \frac{\left[\frac{D}{R} \right]_c^{W,(r_0, \theta_0)}}{\left[\frac{K}{R} \right]_c^{a,ref} Cr^2_{100}} \tag{6}$$

$$\frac{G(r, \theta)}{G(r_0, \theta_0)} F(r, \theta) g(r) = \frac{\left[\frac{D}{R} \right]_c^{W,pos}}{\left[\frac{D}{R} \right]_c^{W,(r_0, \theta_0)}} \tag{7}$$

The right hand side (RHS) of equation (7) describes the dose deposition in the medium normalized to the reference point and it is very obvious that, the result from the energy deposition in the medium is not directly separable into factors as indicated on the left hand side (LHS) of equation (7), providing flexibility in regrouping the factors in a manner to improve the fitting [13].

A modified AAPM TG-43 formalism was used for dose rate calculation as given below:

$$\dot{D}(r, \theta) = S_k \Lambda \frac{G(r, \theta)}{G(r_0, \theta_0)} f(r, \theta) \tag{8}$$

Where

$$f(r, \theta) = g(r) * F(r, \theta) \tag{9}$$

Equation (9) essentially represents the dose rate normalized at the reference point with the effect of the geometry factor also eliminated to reduce the steepness of the dose rate curve, improving the efficiency of fitting thereof.

The formulae by Lliso et al are given below as equation (10) and (11) [4, 6].

$$F(r, \theta) = K(r) + \frac{a(r) \left(\frac{\theta}{\pi} \right)^{e(r)}}{1 + b(r) \left(\frac{\theta}{\pi} \right)^{e(r)}} + \frac{a'(r) \left(1 - \frac{\theta}{\pi} \right)^{e'(r)}}{1 + b'(r) \left(1 - \frac{\theta}{\pi} \right)^{e'(r)}} \tag{10}$$



$$g(r) = \frac{hr^i}{1 + jr^k} \quad (11)$$

Where h , i , j and k are fitted constants from the Monte Carlo data of the source used.

2.2 Functional form for $f(r, \theta)$

Analytically fitted formulae in excel was used to calculate $F(r, \theta)$ and $g(r)$ at points within the vicinity of the HDR ^{192}Ir Nucletron microSelectron “new or v2” design based on the formulation of Lliso’s et al to simulate MC output. This was done to serve as a benchmark for the new fitted parameters in equations (9), (12), (13) and (14). $f(r, \theta)$ values were calculated at both radial distances ($0.25 \text{ cm} \leq r \leq 10 \text{ cm}$) and ($0.25 \text{ cm} \leq r \leq 5 \text{ cm}$) for angles 0° to 180° in 10° intervals using the Microsoft EXCEL spreadsheet. $f(r, \theta)$ exhibits both radial and angular dependence which suggests a strong correlation between $f(r, \theta)$ and the anisotropy function, therefore a statistical variable $\Delta f'$ was introduced to link the two functions as

$$\Delta f' = F(r, \theta) - f(r, \theta) \quad (12)$$

The functional form of the angular component of $F(r, \theta)$ from Lliso et al in equation (10) was retained and equation (12) rearranged to give equation (13) [4, 6];

$$f(r, \theta) = \Delta f + \frac{a(r) \left(\frac{\theta}{\pi}\right)^{e(r)}}{1 + b(r) \left(\frac{\theta}{\pi}\right)^{e(r)}} + \frac{a'(r) \left(1 - \frac{\theta}{\pi}\right)^{e'(r)}}{1 + b'(r) \left(1 - \frac{\theta}{\pi}\right)^{e'(r)}} \quad (13)$$

Where

$$\Delta f = K(r) - \Delta f' \quad (14)$$

Equation (14) consolidates all the linear terms into one function thereby reducing the number of parameters. In essence, Δf was treated as a perturbation and the functional fitting focuses on $\Delta f'$ being appended to the truncated form of the Lliso et al formulation.

2.3 Functional fitting procedure

Given a form that uses A, B, C parameters to accommodate N data points for the fitting process of the tabulated data Δf , an objective function was defined as:

$$F_{obj} = \sum_{i=1}^N W_i [F_{fit}(A, B, C, \dots, \theta_i, r_i) - F_{data}(\theta_i, r_i)]^2 \quad (15)$$

Where W_i is the weighting factor chosen according to the individual datum, $F_{fit}(A, B, C, \dots, \theta_i, r_i)$ is the fit/selected functional form for Δf , (A, B, C, \dots) is the parameters /coefficients, (θ_i, r_i) is the variables of θ and r and F_{data} is the data which the functional form was fitted to for both radial ranges. The unknown coefficients were computed by doing a least squares fit, which minimizes the sum of the squares of the deviations of the data from the model with MatLab backslash operator. The main aim was to find functions that describe the relationship between the variables observed and determining which function will be best used to model the data. Fitting or choosing the appropriate parameters were performed that minimizes the objective function by iteratively varying the parameters using the MatLab curve fitting option to the direction of $R^2 = 1$ and the Root Mean Square Error ($RMSE$) = 0.

The shape of the function Δf for given values of θ was very similar to the curve produced by a transcendental function in an exponential form in equation (16);

$$y = \exp\left(\frac{K}{r}\right) \quad (16)$$

The expression in equation (16) is rotated about the axis $y = kr$, where K and k are positive constants. The final fit functional form made use of as few parameters as possible by combining polynomial of the order 2 and the exponential, in the form described above, to accommodate the data as

$$\Delta f = Ar^2 \exp\left(\frac{a}{r}\right) + Br \exp\left(\frac{b}{r}\right) + C \quad (17)$$

The fitting function in equation (17) provided a better fit which greatly reduced the fitting error than that achieved by other functions. The function was fitted in two separate ranges ($0.25 \text{ cm} \leq r \leq 5 \text{ cm}$) and ($0.25 \text{ cm} \leq r \leq 10 \text{ cm}$) as a user-defined function of the MatLab application to match the data as close as possible and the fit performed using a nonlinear least-squares fit (Levenberg-Marquardt) [14]. Initially the weighting factor of each data were fixed to zero and then adjusted within the range zero to one (0-1).

2.4 Test of the fit functional form

In order to verify the accuracy of the fit functional form, data of dose-rate per unit air kerma



strength ($cGyh^{-1}U^{-1}$) distribution of the source were produced in a polar coordinate form $\dot{D}(r, \theta)$ for comparison purposes. The radial distances used in this study were 0.25, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 cm within ranges ($0.25\text{ cm} \leq r \leq 5\text{ cm}$) and ($0.25\text{ cm} \leq r \leq 10\text{ cm}$) and angular directions at intervals of 10° within the range ($0^\circ \leq \theta \leq 180^\circ$). The coordinate system used had $\theta = 0^\circ$ as the distal end of the source and $\theta = 180^\circ$ as the proximal end.

This was performed using the modified TG-43 formalism with the functional form for $f(r, \theta)$, published dose rate constant and the geometrical factor of the TG-43 parameters. An arbitrary weighting scheme was used in the fit to achieve the highest quality fit possible. Using the fit of Lliso et al output as a benchmark, the deviation of the new fit's dose rate distribution from that of Lliso et al can be considered as errors (E) in the dose distributions [4, 6]. Therefore, the deviations of the mean doses of the new fit from that of Lliso et al were quantified for both ranges using the following relation.

$$E = \text{Fit}_{\text{dose}} - \text{Lliso}_{\text{dose}} \quad (18)$$

3. RESULTS AND DISCUSSIONS

Published data of anisotropy function, $F(r, \theta)$ and Radial dose function, $g(r)$ fitted with functional forms [equation (14) and (15) respectively] utilized 32 parameters and 9 different formulas to compute dose rate distribution. Although a high degree of accuracy was achieved by this method, a lot of task was placed on the computer memory, storage capacity and speed. A relative simple formula used for the fit as indicated in equation (13) utilized 29 parameters and 7 different formulas to produce a more accurate fit with the data used. Clinically, using equation (8) as compared to equation (1) makes dose rate calculations easier (both by hand and by computer) and extremely useful for quick hand check verifications for planning clinical implant. In another breath, interpolating for dose rates with a single table of parameters for $f(r, \theta)$ also produces a clinically more practicable dose rate verification method than the very cumbersome method of combining separate tables for $F(r, \theta)$ and $g(r)$.

3.1 Functional fits for Δf

Fits for Δf values were relatively higher with fit errors less than 0.15% and the maximum discrepancies less than 0.2% for all the angles within the range

($0.25\text{ cm} \leq r \leq 5\text{ cm}$) "data not shown". For the range ($0.25\text{ cm} \leq r \leq 10\text{ cm}$), the average fit discrepancies between the fit and data obtained from the outputs of the Monte Carlo were less than 0.95% and the maximum discrepancies, less than 3.0% over the full range of the fit for all angles. Table 2 shows the results of the fitted parameters of the fit model. The fitting errors in Δf however is not clinically relevant as it does not translate directly to the error in the dose. Rather the error in $f(r, \theta)$ translates directly to the error in dose. The dose rate per unit air kerma strength therefore exhibited the same fitting errors as $f(r, \theta)$. Results of the curve fitting for Δf are plotted in Fig.1 to show data values with corresponding fit values.

Fig. 1a and Fig. 1b show the measured data (dotted) and their corresponding fitted lines in MatLab. The fitted parameters of the data Δf at varying angles are given in Table 2.

3.2 New functional form $f(r, \theta)$

In Lliso's study, the average fit discrepancies between MC data and fit value was around 0.6% for the combined radial dose function and anisotropy function [4]. In comparison to this study, the average fit discrepancies for $f(r, \theta)$ between the data obtained from MC outputs and the fit values were less than 0.3% as shown in Table 3. This shows that the results were marginally better than Lliso's fit. In the clinically relevant range of ($0.25\text{ cm} \leq r \leq 5\text{ cm}$), it was less than 0.04% indicating minimal discrepancies between fit and data values, and should be a justification for further work in this area to build more versatile and clinically more practicable models.

3.3 Dose rate deviation (E)

Dose rate deviations between the MC data and MatLab fits were quantified as error (E) margins and calculated for two separate ranges ($0.25\text{ cm} \leq r \leq 10\text{ cm}$) and ($0.25\text{ cm} \leq r \leq 5\text{ cm}$), where the results showed very minimal error values. The error (E) margin for ($0.25\text{ cm} \leq r \leq 10\text{ cm}$) ranged from -0.091 to 0.137 over the whole range, whilst the margin for ($0.25\text{ cm} \leq r \leq 5\text{ cm}$) ranged from -0.257×10^{-03} to 3.536×10^{-03} "data not shown". This observation suggests the need for further exploitation and concentration on the clinically relevant range for more work in this area.

4. CONCLUSION



Reductions in dose calculation errors provide quality radiobiological data contributing to the improvement of tumour control and reduction in normal tissue complication. The model presently in use internationally for dose calculations is based on the TG-43 report and calculates dose with overall uncertainties around 10 %. The MC method calculates doses to uncertainties of about 2-6 % for brachytherapy. However the resources required for direct MC calculations for routine clinical applications is currently too great. This work presents a closed functional form based on Monte Carlo dosimetry data for mHDRv2 ^{192}Ir brachytherapy source. The function is obtained in a simple and accurate form for easier data handling and use in treatment planning systems to represent MC results with fitting errors well within data uncertainties. The work shows a lot of promise in building robust models to directly represent outputs from contemporary brachytherapy dosimetry calculations to make clinical implementation practicable.

5. RECOMMENDATION(S)

A successful attempt to further reduce the number of parameters, whilst reducing uncertainty and most importantly, customizing the models to directly represent outputs from contemporary brachytherapy dosimetry calculations would be laudable.

6. DEDICATION

This work is dedicated to the memory of the late Mr. E.K. Nani, who supervised the entire research study.

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APPENDIX

Table 1. Characteristics of HDR ¹⁹²Ir brachytherapy source

	MicroSelectron "New or v2" design
Active length	3.6
Distal source face-to-tip distance	0.2
Active diameter	0.65
Total diameter	0.9
Encapsulation	Stainless steel
Manufacturer	Nucletron

*All the physical dimensions are in millimetres (mm)

Table 2. Fitted parameters of the data Δf at various angles (0.25-10 cm)

Angles (°)	Coefficients (with 95% confidence levels)			Constants	
	A (cm ⁻²)	B (cm ⁻¹)	C	a(cm)	b(cm)
0	-0.002291	0.05024	0.1223	1.5910	0.7506
10	-0.002445	0.05064	0.1244	1.5130	0.7263
20	-0.002595	0.05057	0.1285	1.3470	0.6839
30	-0.002663	0.04628	0.1330	-0.2457	0.6457
40	-0.002594	0.04382	0.1378	-0.7699	0.6451
50	-0.002632	0.04424	0.1382	-0.6352	0.6361
60	-0.00278	0.04613	0.1332	-0.4939	0.6397
70	-0.00280	0.04643	0.1327	-0.4446	0.6353
80	-0.002552	0.04356	0.1381	-0.5838	0.6543
90	-0.002754	0.04603	0.1322	-0.4560	0.6489
100	-0.00280	0.04583	0.1334	-0.6380	0.6442
110	-0.002714	0.04528	0.1327	-0.6319	0.6525
120	-0.002649	0.04488	0.1317	-0.6503	0.6642
130	-0.00267	0.04538	0.1326	-0.5337	0.6530
140	-0.002576	0.04488	0.1320	-0.5330	0.6636
150	-0.00262	0.04602	0.1333	-0.2677	0.6466
160	-0.002531	0.05018	0.1289	1.3530	0.6854
170	-0.00236	0.05017	0.1245	1.5320	0.7326
180	-0.002215	0.04997	0.1212	1.6260	0.7654

Table 3. Average percentage error and maximum percentage error of the fit (as defined in the text) for the range data (0.25 cm ≤ r ≤ 10 cm)

r(cm)	Average (%)	Max %
0.25	0.226	0.806
0.5	0.067	0.22
1	0.219	0.382
2	0.078	0.301
3	0.231	0.482
4	0.249	0.487
5	0.17	0.38
6	0.06	0.214
7	0.113	0.201
8	0.199	0.255
9	0.138	0.241
10	0.188	0.332

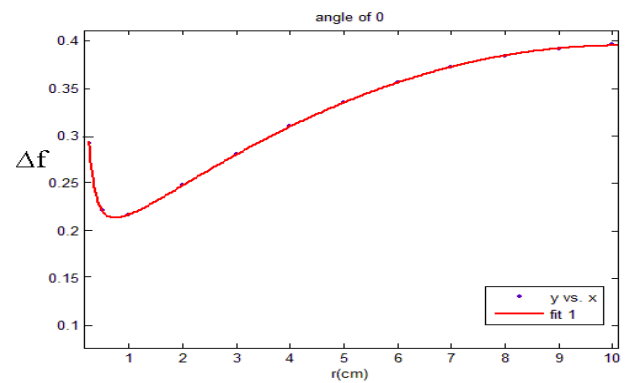


Fig 1a. Curve fitting graph at 0° for Δf_{data} ranging from 0.25-10 cm for mHDRv2 ¹⁹²Ir source to generate the fit data

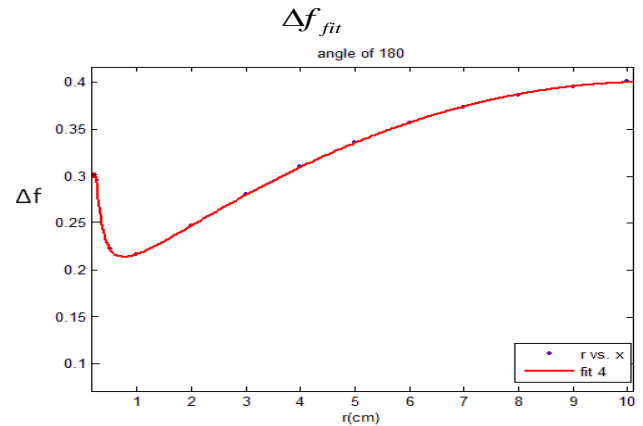


Fig 1b. Curve fitting graph at 180° for Δf_{data} ranging from 0.25-10 cm for mHDRv2 ¹⁹²Ir source to generate the fit data