



Implementation of 3-D Anisotropy Corrected Fast Fourier Transform Dose Calculation around Brachytherapy Seeds

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ABSTRACT

A routine dose computation around brachytherapy seeds employing the Fast Fourier Transform (FFT) has been demonstrated and likened to dose computations in a similar field based on the TG43 recommendations. In applying the FFT convolution model to dose computation, a dose convolution kernel which takes into consideration the anisotropy of the dose distribution around a brachytherapy source in the presence of tissue and applicator heterogeneities was presented. Resulting from the convolution kernel were functions with polynomial and exponential terms. The solution to the convolution integral was thus represented by the Fast Fourier transform. The versatility of the Fast Fourier Transform becomes evident with its speed of dose computation and accuracy evenly matched. It has also been described as ubiquitous; as successful procedures developed in one field could easily be adopted in other fields (E.g. external beam and brachytherapy). In typical dose computation procedures, the dose computation time is usually proportional to the number of sources or dwell positions required for the implants and relatively increases for large number of sources. With a dose convolution kernel developed for a 64x64x64 matrix size with wrap around ordering, the kernel was convoluted with the source distributions also in similar matrix size in 3D. Results from the analysis showed the FFT based convolution method in brachytherapy was comparable in speed with computerized treatment planning system whose model is based on the recommendations of the TG43. Further development on the FFT incorporating techniques such as the Monte Carlo simulation will be required to fine tune the potency of the FFT convolution in studying tumor growth and the aftermath upshots of radiation therapy.

Keywords: *Dosimetry, Monte Carlo Simulation, TG 43, Brachytherapy, Treatment Planning System, Fast Fourier Transform.*

1. INTRODUCTION

The successes chalked in the delivery of radiotherapy fall short if not matched with rigorous dose calculation models and algorithms [1]. This is primarily because the functionality and quality of any treatment planning system (TPS) is heavily dependent on the types of algorithms used in the different stages of the planning process. In modern TPS several different types of dose calculation algorithms are used depending on the manufacturer, but an ultimate dose computational algorithm should be easy to use with its speed and accuracy well juxtaposed. For every good algorithm, the quality of the dose representation is strongly dependent on the data or the parameters used by the algorithm [2]. The agreement between the calculated and delivered dose is of great significance in the radiation therapy

since the accuracy of the absorbed dose as prescribed determines the clinical outcome [3].

The two fold application of dose calculation algorithm in radiation oncology practice; first for the plan optimization in the treatment planning process and secondly for the retrospective analysis of the correlation between treatment parameter and clinical outcome defines two mutually conflicting goals of the respective dose calculation algorithm [3]. First, the dose calculation algorithm has to be fast such that the treatment planning process could be completed in clinically acceptable time frame and secondly the result of the dose calculation has to be sufficiently accurate so that the establishment of correlations between delivered dose and the clinical effects remains reliable and meaningful. This has necessitated a further probe into dose calculation



formalisms and models with the latest being the TG43 report [3].

Dose volume Histograms (DVHs) are significantly utile in anatomy based optimization of dose for both High dose rates (HDR) and Low dose rate (LDR). These DVH's require utmost accuracy since the minutest overdose is enough to offset a clinical complication. However, such procedures are time consuming and tedious especially for large volume implants which may run into several thousands of sampling points [4]. Employing the Fast Fourier Transform (FFT) based convolution method; the time needed for the computation of dose distribution is practically independent of the number of sources. An intrinsic feature of the FFT based convolution is its minimal sample size requirements; of the other of $N \log N$ compared to the discrete Fourier transform whose requirement is of the order of N^2 [4].

Significant errors are prevalent in brachytherapy TPS. These errors results from dose calculation models failing to adequately account for brachytherapy heterogeneities which could perturb the dose up to about 50% [4] and obviously this is required to be accounted for to forefend adverse radiobiological effects such as late fibrosis and fistulae to the rectum and ulcers to the bladder. The FFT based convolution algorithm has been clinically validated in External Beam Radiotherapy (EBRT) for curtailing errors due to heterogeneities [4, 5] and there is a need for further probe and refinement on the FFT-convolution model in brachytherapy practices due to the prominent potential it possesses thereby making it amenable in addressing dosimetry problems due to heterogeneities, speed and anisotropy.

Again the Fast Fourier based convolution calculation is more suited for programming with computers with high specifications [6]. Computers used in TPSs are required to possess high processing and storage capabilities since clinical practices require procedures being abreast with current technologies. An intrinsic value of this FFT-based convolution is a corresponding reduction in computer memory requirement [6].

This work aims at the development and the implementation of an accurate, fast and simple anisotropy corrected FFT dosimetric model based on MATLAB's FFT function and with a great potential of addressing the errors due to heterogeneities in brachytherapy application

2. METHOD

According to the convolution theorem, the Fourier transform of the convolution is the complex product of the individual Fourier transforms. Hence for two sampling points described as functions $f(t)$ and $g(t)$ with corresponding Fourier transforms $F[f]$ and $F[g]$ respectively the convolution is given by [5];

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau \quad (1)$$

The dose distribution at the point x by the conventional model is expressed as

$$D(x) = \int_{-\infty}^{+\infty} g(x')f(x-x')dx \quad (2)$$

Where

$$g(x) = \sum_{i=1}^{N_i} S_i \delta(x-x') \quad (3)$$

Equation (3) describes the geometric distribution of the sources and the associated strength in space described by the Dirac function $\delta(x)$. Equation (2) describes a convolution of the kernel $f(x)$ with the function $g(x)$. A solution to equation (2) could be expressed in the form of an inverse Fourier transform, given by

$$D(x) = F^{-1} \left[\frac{1}{N} \cdot F[g(x)] \cdot F[f(x)] \right] \quad (4)$$

F^{-1} Denotes the inverse Fourier transform and N is the sampling size (expressing in Cartesian coordinates, $N = N_x N_y N_z$). The extension of Eq. (4) in 3D gives the three dimensional dose distributions as

$$D(x) = F^{-1} \left[\frac{1}{N_x N_y N_z} \cdot F[g(x, y, z)] \cdot F[f(x, y, z)] \right] \quad (5)$$

According to the TG43 report, the dose rate at a point is given by equation (6) [7];



$$\dot{D}(r, \theta) = S_k \Lambda \left[\frac{G(r, \theta)}{G(r_o, \theta_o)} \right] g(r) f(r, \theta) \quad (6)$$

The quantity S_k is the air kerma strength of source with Λ expressing the dose rate constant. $G(r, \theta)$ is the geometric function and $g(r)$ is the radial dose function. The anisotropy function is defined by $f(r, \theta)$ which is given by equation (7) [7];

$$f(r, \theta) = \frac{\dot{D}(r, \theta) G(r, \theta_o)}{D(r, \theta_o) G(r, \theta)} \quad (7)$$

If large numbers of seeds are randomly oriented or the degree of anisotropy around a single source is limited, the dose rate contributions can be approximated by the average radial dose rate. By summing the single anisotropic points due to seed sources, it can be averaged over 4π geometry and $f(r, \theta)$ is thus approximated by a single radial function $\phi(r)$ called the 1-D anisotropy function which produces the same results due to its averaging effects. The equation (6) therefore simplifies to the equation (8) below for cylindrically symmetric dose distribution [7]

$$\dot{D}(r) = \frac{1}{4} \int_0^{4\pi} D(r, \theta) d\Omega \quad (8)$$

Where $d\Omega = 2\pi \sin \theta d\theta$

The TG 43 formalism then becomes

$$D(r, \theta) = S_k \Lambda \frac{G(r, \theta)}{G(r_o, \theta_o)} g(r) \phi_{an}(r) \quad (9)$$

Where

$$\phi_{an}(r) = \frac{\int_0^\pi D(r, \theta) \sin \theta d\theta}{2\dot{D}(r, \theta_o)} \quad (10)$$

For distances which are equal to 2-3 times the dimension of the characteristic active source, the geometric factor differs from the inverse square law by less than 1% [6] and the geometry factor is described by the inverse square law as:

$$G(r, \theta) = \frac{1}{r^2} \quad (11)$$

Using the TG43 as a guide Nani et al [8] derived the kernel for brachytherapy based on the 1-D anisotropy function as

$$F = \frac{\Lambda}{r^2} g(r) \phi(r) \quad (12)$$

If large numbers of seeds are randomly oriented or the degree of dose anisotropy around single source is limited, then cylindrical geometry effectively simplifies to a spherical geometry [7]. In order to fit the dose deposition kernel to data points the inverse square law dependence of the kernel was suppressed by evaluating an intermediate function defined by

$$F(r) = g(r) \times \phi(r) \times \Lambda \quad (13)$$

Brachytherapy dosimetry parameters by Perez-Calatayud et al [10] through an ESTRO funded project on the quality control and quality assurance models, served as benchmark data for brachytherapy dose calculations in this study.

Table 1; Dosimetric parameters for microSelectron HDR V2 ^{192}Ir source

r/cm	0.25	0.50	1.00	2.00	3.00	4.00	5.00
$\frac{F(r)}{cGy \cdot h^{-1} \cdot U^{-1}}$	1.2804	1.1158	1.0737	1.0711	1.0733	1.0679	1.0573

The dependence of $F(r)$ on distance r from the source center was obtained as in the table above.

2.1 Development of dose kernel using MatLab simulations

A fit functional form using output from Eq. (13) was performed in this study with reasonable fitting errors using the MatLab software curve fitting application. The MatLab edition used in this study was version 7.8 service pack 1 released in Feb. 12, 2009 with license number 161051, and developed by MathWorks. Through this software package the kernel was obtained as:



$$f(r) = \frac{F(r)}{r^2} = \frac{A}{r^3} \exp\left(\frac{a}{r^2}\right) + \frac{B}{r^3} \exp\left(\frac{b}{r^2}\right) + \frac{C}{r^2} \quad (14)$$

Where A, B, C, a and b are constants. The fitted form of the equation (14) to dosimetric distances away from the source depicted by r/cm in Table 1 was thus obtained with the constants as shown in Table 2.

The resultant kernel out of the equation (14) with constants defined in Table 2 was then applied to 10 HDR v2 ^{192}Ir source arranged in a box in Fig. 1 to simulate two catheters for interstitial brachytherapy.

Table 2: Fitting Parameter

Initial values		Coefficients (with 95% confidence levels)			Final values		Goodness of fit	
a	b	A	B	C	a	b	R^2/cm	RMSE
0.2887	0.8841	0.0374	0.1289	1.032	0.0318	-4.218	0.9992	0.003813

2.2 Simulation of source distribution for the FFT convolution using Matlab and TPS

In simulating the 3D dose calculation, the kernel was generated in the (x, y, z) plane using MatLab. Matrices were constructed for both the dose kernel and the source distribution in a $64 \times 64 \times 64$ matrix arrangement and transformed to perform the FFT calculations.

For a comparative analysis of dose rate computations around source distribution based on the FFT-convolution model, the ^{192}Ir sources used in this work were further simulated by the TPS to determine the dose rate distribution around these sources.

Dose rate peaks around the vicinity of the source distribution orientation shown in Fig.1 was read in planes distal to planes containing the source distribution and in a plane adjacent to planes containing the source distribution in both the MatLab's FFT and TPS simulations. All dimensions in Fig.1 are in centimeters and not drawn to scale

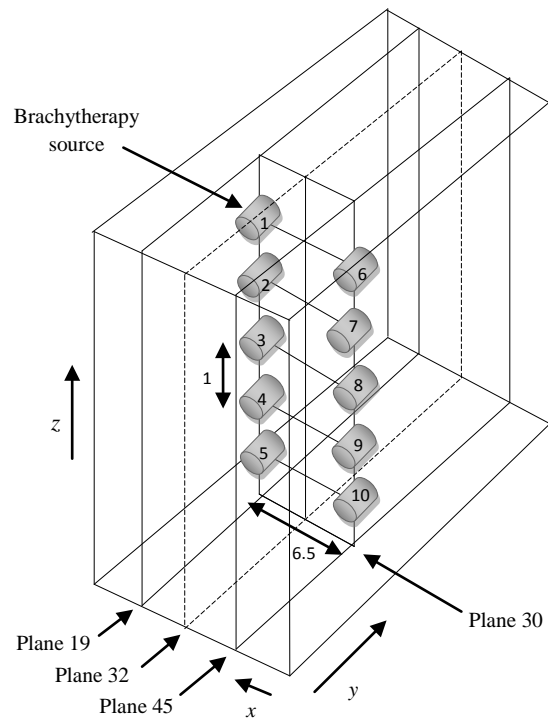


Fig.1: A 3-dimensional sketch of the arrangement of brachytherapy sources



3. RESULTS

The results of dose rate peaks have been obtained from the planes distal to the source planes and the planes adjacent to planes containing the source distribution. Again in a comparative analysis of dose rate peaks from both the FFT convolution and the TPS, graph of dose rate peaks against increasing distances from source center were drawn. The peaks are shown in Figures 2 – 7.

3.1 Dose rate peaks in planes parallel to x-z-plane containing all the sources

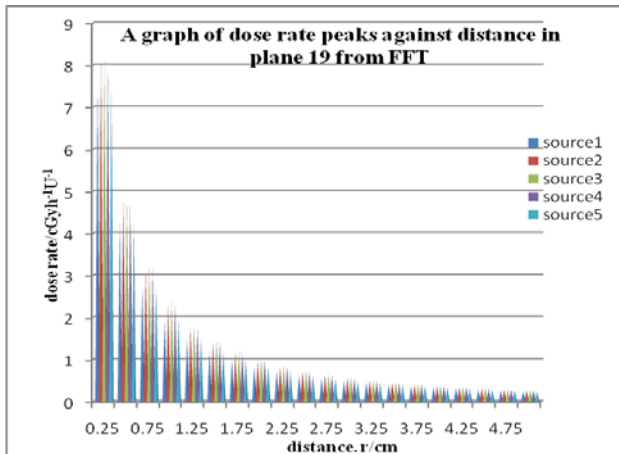


Fig.2: Graph showing dose rate peaks in plane 19 parallel to x-z plane containing all the sources in MatLab's FFT

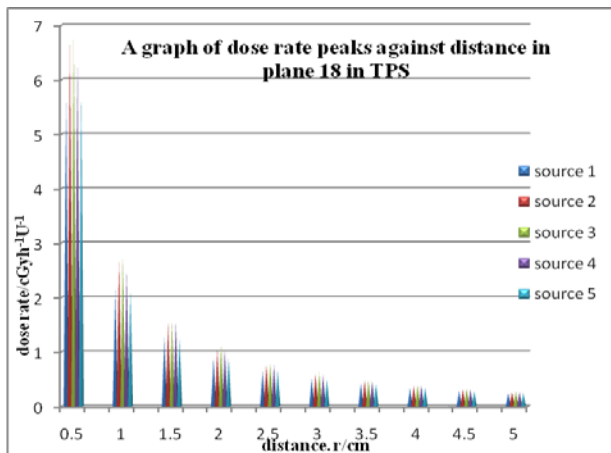


Fig. 3: Graph showing dose rate peaks in plane 19 parallel to x-z plane containing all the sources in TPS

3.2 Dose rate peaks in planes containing the source distribution

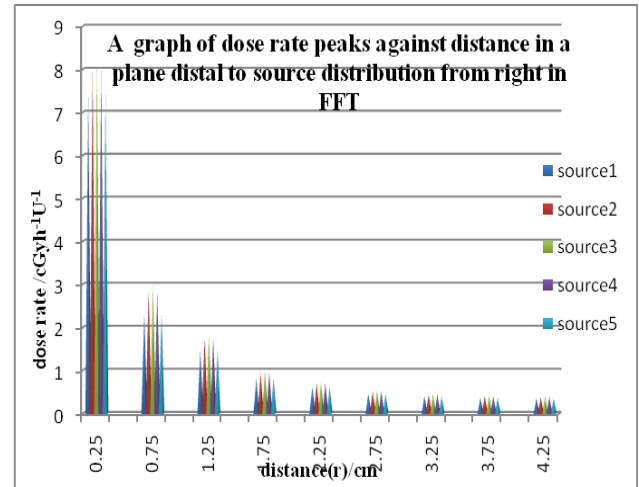


Fig. 4: Graph showing the distribution of dose rate peaks in a plane distal to source distribution from right in FFT

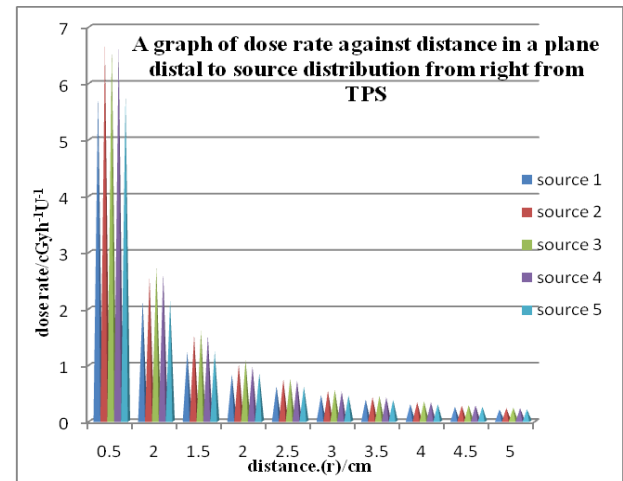


Fig.5: Graph showing the distribution of dose rate peaks in a plane distal to source distribution from the right obtained from TPS



3.3 Dose rate peaks from a plane containing the source distribution within the two sets of sources from FFT

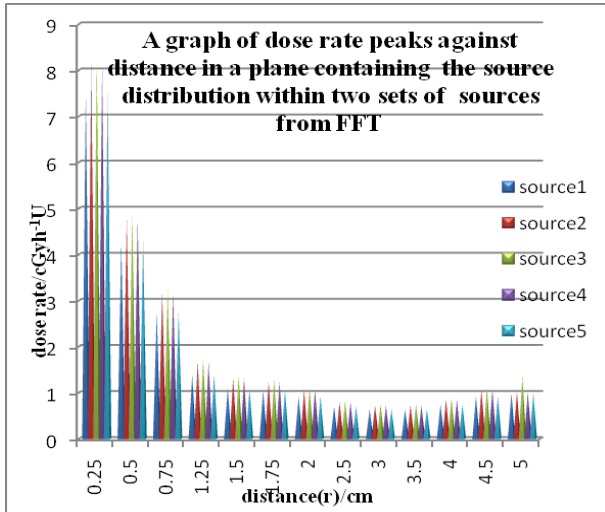


Fig. 6: Graph showing the distribution of dose rate peaks in a plane containing the source distribution within the two sets of sources from FFT

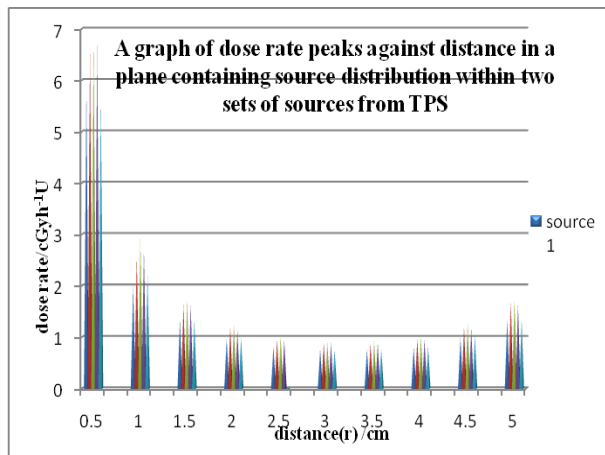


Fig. 7: Graph showing the distribution of dose rate peaks in a plane containing the source distribution within the two sets of sources from the TPS

3.4 Comparison of MatLab’s FFT dose rates with TPS dose rate peaks

The TPS uses a Cartesian coordinate system with origin (0, 0, 0) at the centre of the source. The MatLab’s FFT dose computation on the other hand uses sampling points $N=1, 2, 3, \dots$ etc. The grid size in this work was $0.25 \text{ cm} \times 0.25 \text{ cm} \times 0.25 \text{ cm}$. Hence for a point located in one of the selected planes at distance r_{FFT} in the FFT coordinate system, the equivalent distance which is expected to give the same dose in the TPS coordinate system, assuming all other factors remain the constant was given by

$$r_{TPS} = \sqrt{0.25^2 + 0.25^2 + r_{FFT}^2}$$

In relating r_{TPS} to r_{FFT} , a scale that defines both TPS and FFT dose rates have been drawn as in Table 3.

Table 3: Dose rate peaks of both FFT and TPS at interpolated distance of TPS

TPS Radius (r/cm)	FFT Radius (r/cm)	TPS Dose (cGyh ⁻¹ U)	FFT Dose (cGyh ⁻¹ U)	Error (% Diff)
0.61	0.5	5.2065	4.8733	6.8
1.06	1.0	2.5561	2.3743	7.7
1.54	1.5	1.5324	1.4723	4.1
2.03	2.0	1.0987	1.0152	8.2
2.52	2.5	0.7784	0.7464	4.3
3.02	3.0	0.6138	0.5745	6.8
3.52	3.5	0.4772	0.4577	4.3
4.02	4.0	0.3883	0.3747	3.6
4.51	4.5	0.3246	0.3133	3.6

4. DISCUSSION AND CONCLUSION

The speed and accuracy of dose computation using the FFT convolution is well juxtaposed. The execution of the convolution required a dose deposition kernel defined throughout the entire medium and convoluted with the source distribution to obtain dose rates at a points in a medium. Since the same kernels



must be used throughout the medium there are severe limitations on dose rate anisotropy corrections around the brachytherapy sources. However a formula proposed by Nani et al remedies the situation by taking into consideration the various orientation of the seed source which could be universally applied to all configurations of brachytherapy implants [10]. This corrective model is cumbersome when applied to brachytherapy seeds with their corresponding large number of sources in an encapsulation. In another publication Nani et al [8] suggested the possibility of using a 1D FFT anisotropy correction model for brachytherapy applications but with a strong caution that it should be applied only to specific and the appropriate cases since the practical source was not spherical and might produce significant errors in dose to the critical organs for certain orientation of applicators.

Although analysis in this work was performed on ten sources, several applications of brachytherapy require a large number of sources running into hundreds and the simplified procedure adopted in this method is justified. The 1D anisotropy factor employed in the generation of the dose kernel in this work has potential benefits for brachytherapy dosimetry without compromising accuracy, as for a large number of sources, the corresponding dose distributions are randomly making the degree of anisotropy around a single source meaningless [8]. Also a dose kernel generated from the 1D anisotropy factor does not only simplify the computation process but also enhances the speed of computation once the kernel generated encompasses the totality of the medium.

Due to potentially high hazards associated with HDR brachytherapy, TPS for HDR brachytherapy are usually dedicated to HDR machines. Dedicated TPS like HELAX (developed by Helax TMS) and PLATO (manufactured by Nucletron Bv) take into account specific characteristics of the source, details about encapsulation, anisotropy etc. In the face of limited resources, the model was tested with the brachytherapy module in the Prowess Panther 4.5; (manufactured by Prowess Inc) which is not dedicated to brachytherapy. In particular, the TPS source was a bare source without encapsulation as there are severe limitations of the systems' algorithm compared to brachytherapy dedicated TPS.

TPS and FFT results were compared and a perfunctory look at dose rate peaks incurred from planes in the x-z-plane of Fig.1 containing all the sources in both TPS and FFT were identified to possess high dose rates. Such an occurrence is intemperately anticipated around the source distribution because; dose rate

distributions around sources are governed largely by the inverse square law. It is worthwhile establishing that the dose kernel developed for the FFT convolution also appraises the inverse square law [11]. A further proof of this is established by the graphs of dose rate peaks in Figs 2 and 3 respectively for FFT and TPS where the dose rates decrease with distance in a similar fashion in both systems.

Along the x-z planes between y-z plane 19 and 45 shown in Fig 1, showing source distribution in both FFT and TPS, the mutual influence of sources in the adjacent strands was obvious. Dose rate distributions in such planes initially followed the trend established above viz. The dose rates were observed to reduce with distance from the source, but beyond a certain point however, the dose rates in both the FFT and TPS starts to increase on account of the contributions from the adjacent set of sources, in line with the principles of the basic physics of the interaction of radiation with matter [8].

From Fig 1, anterior-posterior orientation of the sources and its corresponding calculation points along y-z planes distal to x-z plane 30 can be demonstrated. The result in the plane has shown the same trends as the result analyzed in the anterior-posterior orientation of the sources with calculation points along y-z planes 19 and 45. The TPS uses a Cartesian coordinate system with origin (0, 0, 0) at the centre of the source. The MatLab's FFT dose computation on the other hand uses sampling points $N=1, 2, 3, \dots$ etc. The TPS therefore exhibits a central peak in this plane. The FFT however exhibits two peaks in the plane containing the sources and in a plane adjacent to this plane with the dose rate values very close to each other, one being slightly higher on account of the influence of the sources in the adjacent strand.

The encapsulation reduces the dose rate by an amount dependent on the path length through the encapsulation, hence slight differences in the results were expected. Owing to the fact that the TPS sources were bare, high dose rates are gestated with the TPS compared with dose rates from the FFT-convolution. To perform further analysis on the relative magnitudes of the TPS and FFT, a sample plane was selected; a plane containing all the sources. Considering that the TPS uses a Cartesian coordinate system with origin (0, 0, 0) at the centre of the source and the MatLab's FFT dose computation on the other hand uses sampling points possibly $N = 1, 2, 3, \dots$ etc. Taking note of the fact that the finite grid size in this work was 0.25 cm x 0.25 cm x 0.25 cm the inference was the FFT coordinates would not match with the TPS coordinates. The FFT values were read directly from the raw data and the equivalent



TPS values calculated as in Table 3. The dose rate from the TPS that corresponds to these FFT values needed to be interpolated and a method which suppresses the inverse square law dependence on dose rate was used to reduce the steepness of the curve during interpolation, improving the accuracy and efficiency of interpolation.

The discrepancies from the dose rate comparisons ranged between; 3.6 % to 8.2 % showing a general trend of decreasing error with distance. The dose rates from the TPS were consistently higher than the FFT dose rates owing to attenuation from encapsulation of the FFT sources. In an actual deposition of dose by photons, the deposition of dose by secondary electrons depends on the quality of radiation. The quality of radiation from encapsulated sources is harder than the quality from bare sources of the same radionuclide. As the dose rate reduces with distance, the effects of the encapsulation seem to be more pronounced at distances closer to the source where the errors were higher. This is because the softer photons from the bare source expend much more of their energy at short ranges in matter with the harder ones having a relatively longer range.

The TPS is a standard and reliable system for calculating dose. Although the brachytherapy algorithms were simple in the TPS used, it still provides a strong basis for comparison. The expected trend and relative magnitudes of the results obtained suggest strongly that a reliable FFT model has been established. The use of more rigorous testing methods like Monte Carlo data is recommended to consider further developments in the FFT model.

A potential source of error however could be traced to errors prevalent in the simulation process especially loading the images of the source distribution onto the TPS computer from the digitizer. The sensitivity of the TPS digitizer to minute distance change is very high and hence the slightest deviation in measurement simulation impacts generously on the output dose rate. The local variations in the general trend of decreasing percentage errors with distance could be attributed to this. Discontinuities in the trend were observed at 1.06cm, 2.03cm and 3.02cm. The fact that this was only able to bring about a change in the trend of the error at even a relatively small number of points and not the trend in the relative magnitudes of the TPS and FFT results was a confirmation that errors in the TPS calculation were minimal and the results reliable.

The dose rate distributions around seeds calculated by the FFT-convolutions could as well compare with established models such as the Monte Carlo simulations to inform further developments in the FFT-convolution method.

The study focused on the capabilities of the FFT-convolution in dose rate computations. A further analysis based on DVH's could as well be done to investigate the potency in terms of the optimized speed and accuracy of the FFT-convolution. The most common brachytherapy procedures involve tandem and ovoid source. To enrich the affirmation that the FFT-convolution be further refined to take over the TG43 model, there would be an urgent need to continue with studies on such grounds considering tandem source and possibly both tandem and ovoid sources. The technique however could be extended for other brachytherapy seed sources such as the Microselectron iridium brachytherapy sources and Bebig cobalt and cesium brachytherapy sources. Dose Kernels developed from characteristics of such sources could as well be verified by the Monte Carlo simulations to establish a consistency.

5. DEDICATION

This work is dedicated to the memory of the late Mr. E.K. Nani, who supervised the entire research study.

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