Volume 2 No.5, May 2012 ISSN 2224-3577
International Journal of Science and Technology

http://www.ejournalofsciences.org

Solution: The Periodic Ordinary Differential Equation of $4-^{th}$ Order with Direction Functions

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ABSTRACT

In this research, we studied an ordinary differential equation of degree 4-th depending on functions $(f_0 \, and \, f_1)$ vector periodic and proven on existness and uniqueness solution within domain R^m.

Keywords: Differential equations

1. INTRODUCTION

In this research, f_0 and f_1 vector periodic in normal high order differential equations of order 4-th with high frequency that a specific conditions are put in this research is differ from (Bateman H. 1985, Boyce W.E., Di Prima R.C. 1977). Whom studied a normal differential equation of order one for trigonometric function. While (Greenberg M.D.1994 and struble R.A) dealt with study of differential equation of order two for vector periodic function during which they put a specific conditions. In [7], the aim is to explain some conditions for nonlinear differential equation $\dddot{x}+\omega^2 x+\mu f(x,\dot{x},\ddot{x})=0$, in order to have periodic solutions. This kind of differential equation occurs in many mechanical systems such as vibration system and many physical engineering problem; specially in energy and acceleration problems.

Suppose that :

$$f(x,y,z)=\frac{\partial F(x,y)}{\partial x}y+\frac{\partial F(x,y)}{\partial y}z \text{, where}$$

$\dot{x}=y$, $\ddot{x}=z$ and $F(0)=DF(0)=0$; then it could be proved that if the second differential equation $\ddot{x}+\omega^2 \dot{x}+\mu F(x,\dot{x})=k$ has periodic solutions, so the original equation has too. Therefore they reduced the third differential equation to the second differential equation. Here they considered periodic solution for the above second differential equation. They would prove that under some condition on partial derivative of the function f at the fixed point, many periodic solutions occur for the third differential equation as the parameter k varies. In [8], proved multiplicity of small amplitude periodic solutions, with fixed frequency ω, of completely resonant wave equations with general nonlinearities. As $\omega \to 1$ the number N_ω of $\frac{2\pi}{\omega}$-periodic solutions $u_1,\ldots,u_n,\ldots,u_{N\omega}$ tends to $+\infty$. The minimal periodic of the $n\,th$ solution u_n is $\frac{2\pi}{n\omega}$. In [9], considered the existence, multiplicity and nonexistence of positive ω-periodic solutions for the periodic equation
$x'(t)=a(t)g(x)x(t)-\lambda b(t)f(x(t-\tau(t)))$, where $a,b\in C(\mathbf{R},[0,\infty))$ are ω-periodic,

$$\int_0^\omega a(t)dt>0,\ \int_0^\omega b(t)dt>0,\ f,g\in C([0,\infty),[0,\infty)),$$

and $f(u)>0$ for $u>0, g(x)$ is bounded, $\tau(t)$ is a continuous ω-periodic function. Define $f_0=\lim_{u\to 0^+}\frac{f(u)}{u}$, $f_\infty=\lim_{u\to\infty}\frac{f(u)}{u}$, $i_0=$number of zeros in the set $\{f_0,f_\infty\}$ and $i_\infty=$number of infinities in the set $\{f_0,f_\infty\}$.

They showed that the equation has i_0 or i_∞ positive ω-periodic solutions for sufficiently large or small $\lambda>0$, respectively. In [10], studied a spectrally accurate numerical method for finding non- trivial time periodic solutions of non-linear partial differential equations. The method is based on minimizing a functional (of the initial condition and the period) that is positive unless the solution is periodic, in which case it is zero. They are solved an adjoint PDE to compute the gradient of this functional with respect to the initial condition. They are included additional terms in the functional to specify the free parameters, which , in the case of the Benjamin- Ono equation, are the mean, a spatial phase, a temporal phase and the real part of one of the Fourier modes at t=0. In [11], work the method of asymptotic integration of the singular perturbed nonlinear system of differential equations is suggested, studied the system of equations

$$\varepsilon \frac{dx}{dt} = A(t,\varepsilon)x + f(t,\varepsilon,x), \qquad x(0,\varepsilon) = x_0,$$

where $\varepsilon(0 < \varepsilon \le \varepsilon_0)$ is a small parameter, $f(t,\varepsilon,x),\ x(t,\varepsilon), x_0$ is n-dimensional vectors, and supposed to carry some conditions. While in this research they worked studying a differential of order 4-th for any two vector periodic function so by this work they considered this study is an important and expand study.

2. STATEMENT OF THE PROBLEM

Let m and p-natural number, and also even, $p = 2$, and $G_i = 0,1,2$ organic domain in space R^m .We have to study problem $2\pi\omega^{-1}$ periodic solution for differential equations to be fourth degree.

$$\frac{d^4u}{dt^4} = f_o\left(u, \frac{du}{dt}, \frac{d^2u}{dt^2}, \omega t\right) + \omega^2 f_1(u,\omega t) \qquad (1)$$

where ω- big parameter. We will be presuppose the following:

1. Vector functions $f_o(e,\tau)$ defined in the set $\Omega_o = \{e,\tau, e \in G_o \times G_1 \times G_2, \tau \in R\}$ u vector functions $f_1(u,\tau)$ defined in the set $\Omega_1 = \{u,\tau,u \in G_o, \tau \in R\}$, have meaning in R^m .
2. Vector functions $f_o(e,\tau)$ and $f_1(u,\tau)$ have continuously differentiable for any order with respect to e and u respectively.

Asymptotic expansion solution equation (1) will be sought in the from

$$u_\omega(t) = \sum_{j=0}^{\infty} \omega^{-j} u_j + \sum_{j=2}^{\infty} \omega^{-j} v_j(\omega t)$$

Where $v_j(\tau) - 2\pi$ periodic vector functions have meaning in R^m .

u_j -vector in R^m and

$$\langle v_j \rangle = \frac{1}{2\pi} \int_0^{2\pi} v(\tau)\, d\tau = 0 \qquad (2)$$

We substitute equation (1) in place of $u, \frac{du}{dt}, \frac{d^2u}{dt^2}$ expression (2) and we develop nonlinear f_0 and f_1 in Taylor series, as a result we have the following equation:

$$\sum_{j=2}^{\infty} \omega^{-j+4} \frac{\partial^4 v_j}{\partial \tau^4} = f_0\left(u_0, 0, \frac{\partial^2 v_2}{\partial \tau^2}, \tau\right) + \frac{\partial f_0\left(u_0, 0, \frac{\partial^2 v_2}{\partial \tau^2}, \tau\right)}{\partial e_0} \left[\sum_{j=1}^{\infty} \omega^{-j} u_j + \sum_{j=2}^{\infty} \omega^{-j} v_j \right] +$$
$$\frac{\partial f_0\left(u_0, 0, \frac{\partial^2 v_2}{\partial \tau^2}, \tau\right)}{\partial e_1} \sum_{j=2}^{\infty} \omega^{-j+1} \frac{\partial^2 v_j}{\partial \tau^2} + f_0\left(u_0, 0, \frac{d^2 v_2}{d\tau^2}, \tau\right) \sum_{j=2}^{\infty} \omega^{-j+2} \frac{\partial^2 v_j}{\partial \tau^2} + \omega^2 \{ f_1(u_0,\tau) +$$
$$\frac{\partial f_1(u,\tau)}{\partial u} \left[\sum_{j=1}^{\infty} \omega^{-j} u_j + \sum_{j=1}^{\infty} \omega^{-j} v_j \right] + \frac{1}{2!} \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \left[\sum_{j=1}^{\infty} \omega^{-j} u_j + \sum_{j=2}^{\infty} \omega^{-j} v_j \right]^2 \qquad (3)$$

Where

$$\frac{1}{2!} \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2} \left[\sum_{j=1}^{\infty} \omega^{-j} u_j + \sum_{j=2}^{\infty} \omega^{-j} v_j \right]^2 =$$
$$\frac{1}{2!} \sum_{k,s=1}^{4} \left[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k \partial u_s} [\sum_{j=1}^{\infty} \omega^{-j} u_{j_k} + \sum_{j=2}^{\infty} \omega^{-j} v_{j_k}][\sum_{j=1}^{\infty} \omega^{-j} u_{j_s} + \sum_{j=2}^{\infty} \omega^{-j} v_{j_s}] \right]$$

ISSN 2224-3577
International Journal of Science and Technology

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Equations coefficient with positive degree keep in mind:

$$\omega^2 : \frac{\partial^4 v_2}{\partial \tau^4} = f_1(u_0, \tau) \tag{4}$$

$$\omega^1 : \frac{\partial^4 v_3}{\partial \tau^4} = \frac{\partial f_1(u_0, \tau)}{\partial u} u_1 \tag{5}$$

$$\omega^0 : \frac{\partial^4 v_4}{\partial \tau^4} = \frac{\partial f_1(u_0, \tau)}{\partial u} u_2 + \frac{1}{2!} \frac{\partial^2 f_1(u_0, \tau)}{\partial u^2} u_1^2 \tag{6}$$

$$\omega^1 : \frac{\partial^4 v_3}{\partial \tau^4} = \frac{\partial f_1(u_0, \tau)}{\partial u} u_1 + \frac{1}{2!} \frac{\partial^2 f_1(u_0, \tau)}{\partial u^2} u_1^2 \tag{7}$$

The equation (4), where u_0 be considered parameter, it is well known have unique satisfying condition $\langle v_2(\tau) \rangle = \frac{1}{2\pi} \int_0^{2\pi} v(s) ds = 0$.

We produce this solution in the from:

$$v_2(\tau) = \varphi_2(u_0, \tau) \tag{8}$$

Analogous we have the solution equations (5)-(7) with zero mean:

$$v_s(\tau) = \frac{\partial \varphi_2(u_0, \tau)}{\partial u} u_{s-2} + F_s \quad s = 3, \cdots, 5 \tag{9}$$

Where F_s -expression depending on $u_i, 1 \le i \le s-3$. Now we equate in (3) coefficient expansion with ω^0

$$\frac{\partial^4 v_j}{\partial \tau^4} = f_0(u_0, 0, \frac{\partial^2 v_2}{\partial \tau^2}, \tau) + \frac{\partial f_1(u_0, \tau)}{\partial u}(u_2 + v_2) \tag{10}$$

If we substitute expression v_2 in equation (10) and from average, we have the equation :

$$\Phi(u) = \left\langle f_0(u_0, 0, \frac{\partial^2 \Psi_2(u, \tau)}{\partial \tau^2}, \tau) + \frac{\partial f_1(u, \tau)}{\partial u} \Psi_2(u, v) \right\rangle \tag{11}$$

We will presuppose, the equation (11) has stationary solution $u = u_0$, that mean, for vector function $\Phi(u)$ this equation is correct

$$\Phi(u_0) = 0 \tag{12}$$

ISSN 2224-3577
International Journal of Science and Technology

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Where $\Phi(u_0)$ - invertible matrix. Here

$$\Phi'(u_0) = \left\langle \begin{matrix} \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_0} + \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_2} \frac{\partial^3\Psi_2(u_0,\tau)}{\partial\tau^2\partial u} + \\ \frac{\partial f_1(u_0,\tau)}{\partial u}\frac{\partial\Psi_2(u_0,\tau)}{\partial u} + \frac{1}{2!}\sum_{i,k=1}^{4}[\frac{\partial^2 f_1(u_0,\tau)}{\partial u_i \partial u_k}]\Psi_2(u_0,\tau) \end{matrix} \right\rangle \quad (13)$$

Equation coefficient at ω^{-1}, have to equation:

$$\frac{\partial^4 v_5}{\partial\tau^4} = \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_0} u_1 + \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_1}\frac{\partial v_2}{\partial\tau} +$$
$$\frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_2}\frac{\partial^2 v_3}{\partial\tau^2} + \frac{\partial f_1(u_0,\tau)}{\partial u}(u_3 + v_3) + \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1 v_2) \equiv$$
$$\Lambda_3(u_0+\tau) + \frac{\partial f_2(u_0,\tau)}{\partial u}u_3, \quad (14)$$

Here

$$\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_1 v_2) = \sum_{s,k=1}^{4}\frac{\partial^2 f_1(u_0,\tau)}{\partial u_k \partial u_s}(u_{1s} v_{2k}).$$

If we apply the equation (14) operation average with regard (9) we obtain:

$$\left\langle \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_0} \right\rangle u_1 + \left\langle \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_1}\frac{\partial\Psi_2}{\partial\tau} \right\rangle +$$
$$\left\langle \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2}, \tau)}{\partial e_2}\frac{\partial^3\Psi_2(u_0,\tau)}{\partial\tau^2\partial u} \right\rangle u_1 + \left\langle \frac{\partial f_1(u_0,\tau)}{\partial u}\frac{\partial\Psi_2(u_0,\tau)}{\partial u} \right\rangle u_1$$
$$+\left\langle \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\Psi_2(u_0,\tau) \right\rangle u_1 = 0$$

$$\Phi'(u_0)u_1 = -\left\langle \frac{\partial f_0(u_0, 0, \frac{\partial^2\Psi_2(u_0,\tau)}{\partial e_1}, \tau}{\partial e_1}\frac{\partial\Psi_2}{\partial\tau} \right\rangle \quad (15)$$

By virtue equation (9), the solution problem (14) have the meaning:

$$v_5(\tau) = \frac{\partial \Psi_2(u_0,\tau)}{\partial u} u_3 + \chi_3(u_0,\tau) \qquad (16)$$

$\langle \chi_3 \rangle = 0$ and $\frac{d^4\chi_3}{d\tau^2} = \Lambda_3(u_0,\tau)$, where χ_3-expression, depending on u_{s1} and v_{s2} at $0 \le s_1 \le 1$ and $2 \le s_2 \le 3$. From (15) we have define u_1. From (9) we will find v_3:

$$v_3(\tau) = -\frac{\partial \Psi_2(u_0,\tau)}{\partial u}\left[\Phi'(u_0)\right]^{-1}\left\langle \frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau}{\partial \tau^2},\tau}{\partial e_1}\frac{\partial \Psi_2}{\partial \tau}\right\rangle \qquad (17)$$

The equations from coefficients v_j, $j \ge 4$ bear in mind:

$$\omega^{-2}: \frac{\partial^4 v_6}{\partial \tau^4} = \frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}u_2 + \frac{1}{2!}\frac{\partial^2 f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^2}u_1{}^2 +$$
$$\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}v_2 + \frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_1}\frac{\partial v_3}{\partial \tau}$$
$$+\frac{\partial f_0(u_0,0,\frac{\partial\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_2}\frac{\partial^2 v_4}{\partial\tau^2} + \frac{\partial f_1(u_0,\tau)}{\partial u}(u_4+v_4) + \frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_2 v_2) +$$
$$\frac{\partial^3 f_1(u_0,\tau)}{\partial u^3}(u_1^2 v_2) \equiv \Lambda_4(u_0+\tau) + \frac{\partial f_1(u_0,\tau)}{\partial u}u_4$$

$$\omega^{-3}: \frac{\partial^4 v_7}{\partial \tau^4} = \frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}u_3 + \frac{\partial^2 f_0(u_0,o,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^2}u_1 u_2 +$$
$$\frac{1}{3!}\frac{\partial^3 f_0(u_0,o,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^3}u_1{}^3 \equiv \Lambda_5(u_0+\tau) + \frac{\partial f_2(u_0,\tau)}{\partial u}u_5$$

We show that description it is possible find any coefficients expansion (2). Presuppose, that we know $v_2, v_3, \cdots, v_{3+j}$ and $u_0, u_1, \cdots, u_{j-1}$.

We can't find v_{4-j} and u_j. We have equation:

$$\omega^{-j}:\frac{\partial^4 v_{4+j}}{\partial\tau^4}=\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}u_j+\frac{1}{2!}\frac{\partial^2 f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^2}\sum_{m+4=j}u_m u_4$$

$$+\cdots+\frac{1}{j!}\frac{\partial^j f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0^j}u_1{}^j+\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_{2-s}}\frac{\partial^{2-s}v_{2+j-s}}{\partial\tau^{2-s}}+$$

$$\frac{\partial f_1(u_0,\tau)}{\partial u}(u_{2+j}+v_{2+i})+\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}(u_j v_2)+\cdots+\frac{1}{(j+1)!}\frac{\partial^{j+1}f_1(u_0,\tau)}{\partial u^{j+1}}(u_1{}^j v_2)-$$

$$-\left\langle\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}\right\rangle u_j-\cdots-\left\langle\frac{1}{j!}\frac{\partial^j f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^j}\right\rangle u_1^j-$$

$$-\left\langle\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_{2-s}}\frac{\partial^{2-s}v_{2+j-s}}{\partial\tau^{2-s}}\right\rangle-\left\langle\frac{\partial f_1(u_0,\tau)}{\partial u}v_{2+j}\right\rangle-\cdots-$$

$$-\left\langle\frac{1}{(j+1)!}\frac{\partial^{j+1}f_1(u_0,\tau)}{\partial u^{j+1}}(u_1{}^j v_2)\right\rangle \tag{18}$$

for v_{4+j} we get equation :

$$v_{4+j}(\tau)=\frac{\partial\Psi_2(u_0,\tau)}{\partial u}u_j+\chi_j(u_0,\tau) \tag{19}$$

Where χ_j - expression, depending on u_{r_1} and v_{r_2} at $0\le r_1\le j-1$ and $2\le r_2\le 3+j$. Coefficient $u_j, j\ge 1$ are solution linear problem:

$$\left\langle\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0}\right\rangle u_j+\frac{1}{2!}\left\langle\frac{\partial^2 f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^2}\sum_{m+4=j}(u_m u_4)\right\rangle+\cdots+$$

$$\frac{1}{j!}\left\langle\frac{\partial^j f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_0{}^j}u_1{}^j\right\rangle+..+\left\langle\sum_{s=0}^{j}\frac{\partial f_0(u_0,0,\frac{\partial^2\Psi_2(u_0,\tau)}{\partial\tau^2},\tau)}{\partial e_{2-s}}\frac{\partial^{2-s}v_{2+j-s}}{\partial\tau^{2-s}}\right\rangle+$$

$$\left\langle\frac{\partial f_1(u_0,\tau)}{\partial u}v_{2+j}\right\rangle+\left\langle\frac{\partial^2 f_1(u_0,\tau)}{\partial u^2}\Psi_2(u_0,\tau)\right\rangle u_j+\cdots+\frac{1}{(j+1)!}\left\langle\frac{\partial^{j+1}f_1(u_0,\tau)}{\partial u^{j+1}}(u_1{}^j v_2)\right\rangle=0$$

(From here, it is necessary equation)

$$\Phi'(u_0)u_j = M_j \qquad (20)$$

Where M_j -expression as type that χ_j. We consider question about decidability constructed problems. Average problem (12) by condition u_0. Substituting it in expression (8) we find $v_2(\tau)$. After that definable uniquely solution u_1 linear problems (20) with $j = 1$ and by formula (9) at $s = 3$ we can find v_3 and etc.

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