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Analyzing Some Behavior of a Cracked Beam under Pressure

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ABSTRACT

In this paper, a simple cracked beam under vertical pressure is considered. Stress intensity factor, normal, and shear stresses (at the tip of the crack) are calculated with finite element software. Results are compared when there is contact between crack surfaces with different direction of pressure, and when there is no contact. Results show that stress intensity factors can be implemented for predicting the maximum stresses behavior of the beam.

Keywords: *Beam, Crack, Finite element, Pressure, Stress intensity factor*

I. INTRODUCTION

Fracture Mechanics as we know it, was originated by Wieghardt and Inglis[1]. Both independently showed that cavities and flaws in continuum materials act as stress concentrators which, in the limit of sharp edges (cracks), produce infinite stress at the tip[2]. A fairly thorough description of the approaches for solving the crack problems is made by many researchers[3-6]. These were the first attempts to bring closer the theories of fracture mechanics (FM) and continuum mechanics (CM). About the same time, the Finite Element Method (FEM) and digital computers dashed into the engineering community as a gifted means for quantifying solutions in structural and solid mechanics. Naturally, fracture mechanic researchers implemented their FE methods, while continuum mechanic researchers implemented theirs[7].

The rapid development in computing technologies, especially with respect to increased computational power and data storage capacity, has made numerical simulation of crack closure more and more feasible, provided that finite element (FE) models can be shown to be correct and their limitations and applicability are understood[8]. Through performing a FE analysis, there is also the possibility of checking and refining some fundamental assumptions imposed in analytical methods, e.g., the assumption of infinite plates and simplification of material constitutive relations. Research on investigating problems of crack propagation using the FE method commenced in the early seventies[9-10].

The finite elements method can be easily implemented for beam elements without cracks since the stiffness and generalized geometrical stiffness matrixes of a noncracked beam are already commonly known (for example in[11]). However, the situation essentially changes if the structural elements are transversely cracked. The finite element method is a numerical method that can be used for the accurate solution of complex engineering problems. The method was first developed in 1956 for the analysis of aircraft structural problems. Thereafter, within a decade, the potentialities of the method for the solution of different types of applied science and engineering problems were recognized[12]. Over the years, the finite element technique has been so well established that today it is considered one of the best methods for solving a wide variety of practical problems efficiently. In fact, the method has become one of the active research areas for applied mathematicians. One of the main reasons for the popularity of the method in different fields of engineering is that once a general computer program is written, it can be used for the solution of any problem simply by changing the input data[13].

Nowadays, we live a curious situation. On one hand, most structural engineers and FE codes for computational solid mechanics are decanted. On the other, the observed mesh-size and mesh-bias dependence exhibited by these models make the academic world very suspicious about this format. Hence, a lot of effort has been spent in the last 30 years to investigate and remedy the observed drawbacks of this approach[14].

In some complicated problems such as crack and contacts that analytical solution disables to solve them, numerical analysis strongly recommended. Finite element analysis is one of the usual ways to solve this kind of problems.

Stress intensity factor (K) describes the magnitude of the elastic crack tip stress field. In addition, K can be used to describe crack growth and fracture behavior of materials if the crack tip stress field remains predominantly elastic. This correlating ability makes the stress intensity factor an extremely important fracture mechanics parameter[15].

Different beam support conditions and different direction of force that is applied may cause some changes in stress distribution around a crack tip and stress intensity factor. Some different conditions are considered for a simple cracked beam that is show in Fig. 1. Stress intensity factor and stress distributions are calculated and compared with each other, when there is either contact or no contact between crack surfaces

II. MATERIALS AND METHODS

Fig. 1 shows a Simple beam that has a crack inside it. The length, height and crack length of the beam are L, h and a respectively.

If there is a load on surfaces of the beam, there are three independent kinematic movements of the upper and lower crack surfaces with respect to each other. These three basic modes of deformation are illustrated in Fig. 2, which presents the displacements of the crack surfaces of a local element

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containing the rack front. Any deformation of the crack surfaces can be viewed as a superposition of these basic deformation modes[16], which are defined as follows:

a) **Opening mode, I.** The crack surfaces separate symmetrically with respect to the planes xy and xz.
b) **Sliding mode, II**. The crack surfaces slide relative to each other symmetrically with respect to the plane xy and skew-symmetrically with respect to the plane xz.
c) **Tearing mode, III.** The crack surfaces slide relative to each other skew-symmetrically with respect to both planes *xy* and *xz*.

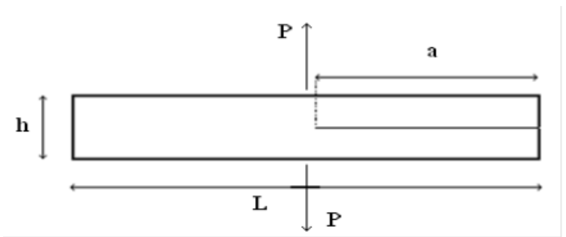

Fig. 1: A Simple Cracked Beam

The stress and deformation fields associated with each of these three deformation modes will be determined in cases of plane strain or plane stress.

The stress intensity factor belongs to each of these cases display as K_I, K_{II}, K_{III} respectively.

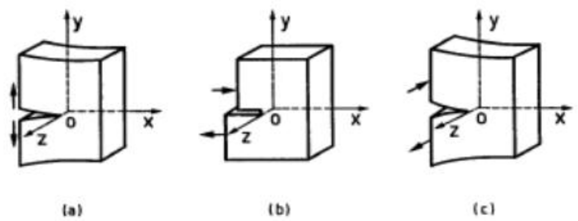

Fig. 2: Three basic modes of crack extension. (a) Opening mode, *I*, (b) Sliding mode, *II*, and (c) Tearing mode, *III*

For single-edge-cracked plate under uniform tension as illustrate in Fig. 1, the formula for calculating of stress intensity factor is as follows[17]:

$$K_I = \frac{P}{B\sqrt{L}}\left(\frac{\sqrt{2\mathrm{Tan}\left[\frac{\pi a}{2L}\right]}}{\mathrm{Cos}\left[\frac{\pi a}{2L}\right]}\left(0.752 + 2.02\left(\frac{a}{L}\right) + 0.37\left(1 - \mathrm{Sin}\left[\frac{\pi a}{2L}\right]\right)^3\right)\right) \quad (1)$$

Where ***P*** is tension force and ***B*** is the beam thickness.

Eq. (1), gives approximate solution of K_I, where the other parameters are unknown like as crack radius (***r***).

Eqs. (2) – (3), can be used when some other parameters are available. For calculating stress intensity factors, Eq. (2) – (3) are more accurate than Eq. (1)[18].

$$K_I = \sqrt{2\pi}\,\frac{G}{1+k}\,\frac{\Delta v}{\sqrt{r}} \quad (2)$$

$$K_{II} = \sqrt{2\pi}\,\frac{G}{1+k}\,\frac{\Delta u}{\sqrt{r}} \quad (3)$$

$$k = \begin{cases} 3 - 4v & \textit{Plane stress} \\ (3 - v)\,/\,(1 + v) & \textit{Plane strain} \end{cases}$$

Δv and Δu, are the motions of one crack surface with respect to the other, v = Poisson's ratio, G = shear modulus, r = crack tip radius.
Some parameters are considered for the beam of Fig. 1:

$\boldsymbol{L} = 1$ meter (*m*), $\boldsymbol{h} = 0.1$ *m*, $\boldsymbol{a} = 0.5$ *m*, thickness ***B*** = 0.1 *m*, Module of elasticity $\boldsymbol{E} = 2 \times 10^{12}\ N/m^2$, $\boldsymbol{v} = 0.3$ and pressure applying to surfaces $\boldsymbol{P} = 10^6\ N/m^2$.

From Eq. (1), $\boldsymbol{K_I} = 3.543 \times 10^6$.

Eq. (2) – (3) need some parameter that must be obtained. Radius of crack tip can be assumed and displacement of crack surfaces can be calculated from some numerical solutions such as finite element method.

K_I, can be calculated directly from finite element method, which is mentioned below. So there can be a comparison between various results of K_I , that obtained from different method.

Finite element approximations are usually quite bad unless a very dense mesh consisting of numerous small elements is modeled around the crack tip. This may not prove feasible at times, and is highly inefficient when computational resources are limited. It will be a better option to model such a problem with what is known as crack tip elements, sometimes known as *singularity elements.*

From theories of linear elastic fracture mechanics, the stresses near the crack tip are characterized by the stress intensity factor, K_I, in Mode *I* fracture as[19]:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{xy}\\ \sigma_{yy}\end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\,\mathrm{Cos}\,\frac{\theta}{2}\begin{bmatrix}1 - \mathrm{Sin}\,\frac{\theta}{2}\,\mathrm{Sin}\,\frac{3\theta}{2}\\ \mathrm{Sin}\,\frac{\theta}{2}\,\mathrm{Sin}\,\frac{3\theta}{2}\\ 1 + \mathrm{Sin}\,\frac{\theta}{2}\,\mathrm{Sin}\,\frac{3\theta}{2}\end{bmatrix} \quad (4)$$

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and the displacement near the crack tip is expressed as[19]:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{K_I\sqrt{r}}{2G\sqrt{2\pi}} \begin{bmatrix} \mathrm{Cos}\,\frac{\theta}{2}\left(k - 1 + 2\,\mathrm{Sin}^2\frac{\theta}{2}\right) \\ \mathrm{Sin}\,\frac{\theta}{2}\left(k + 1 - 2\,\mathrm{Cos}^2\frac{\theta}{2}\right) \end{bmatrix} \quad (5)$$

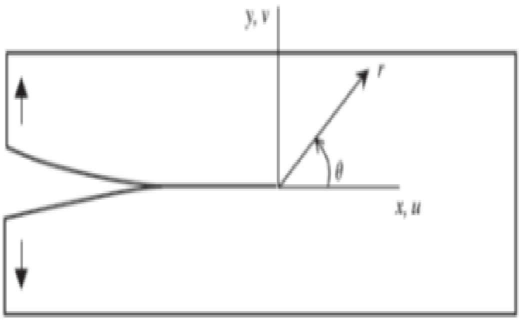

Fig. 3: Crack Tip Position

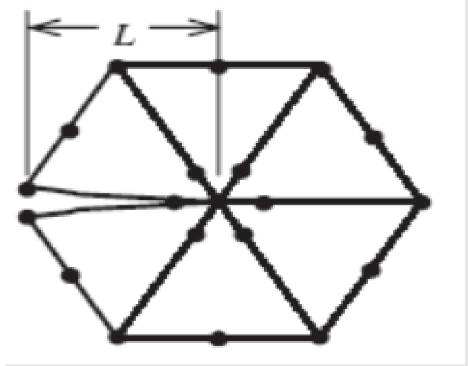

Fig. 5: Triangular crack tip

Fig. 3 is shown r and θ in Eq. (4) – (5)

To approximate the behavior of the stresses and displacements near the crack tip according to the theories of fracture mechanics, a special eight-nodal, quadratic, isoparametric element as shown in Fig. 4 can be formulated. Fig. 5 shows a triangular crack tip element that is more current.

With the aid of interpolation and shape function, displacement and stresses could be obtained. More information is explained in [13,19-21]

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Choose $\boldsymbol{r}$ = 0.25 *mm*, from finite element method, Δv is calculated from crack surface points at $\boldsymbol{r}$ position. Δv = 1.1181×10^{-3} mm.

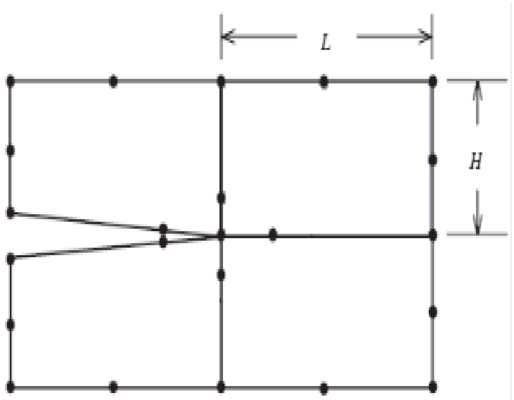

Fig. 4: Modeling of crack tip with crack tip elements

From Eq. (2) and the new obtained parameters:

K_I = 4.86947×10^{7} for *Plane stress*, and K_I = 4.43122×10^{7} for *Plane strain.*

Using finite element software for calculating K_I directly is necessary, because for different beam conditions, it is time consuming to construct beam stiffness element matrix and crack elements, and in some conditions, it is more difficult. ANSYS software is one of the best software for this purpose.

Crack element that is modeled in ANSYS is shown in Fig. 6; *PLANE* element is considered for modeling[22] and from different *PLANE* element in ANSYS software *PLANE82* is appropriate for this model[23]. Fig. 7 shows stress distribution near the crack tip after applying pressure.

Stress intensity factors that are directly obtained from ANSYS software are as follows:

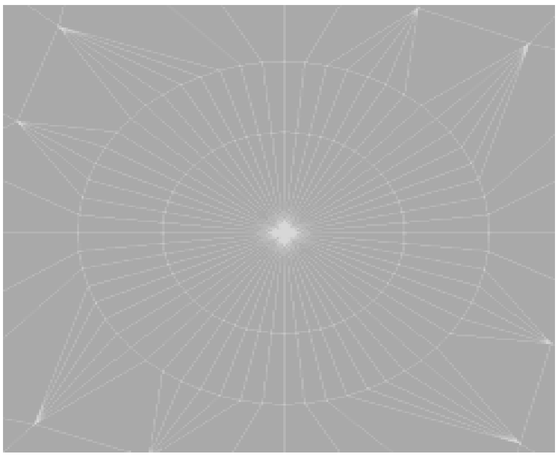

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Fig. 6: crack elements modeled in ANSYS

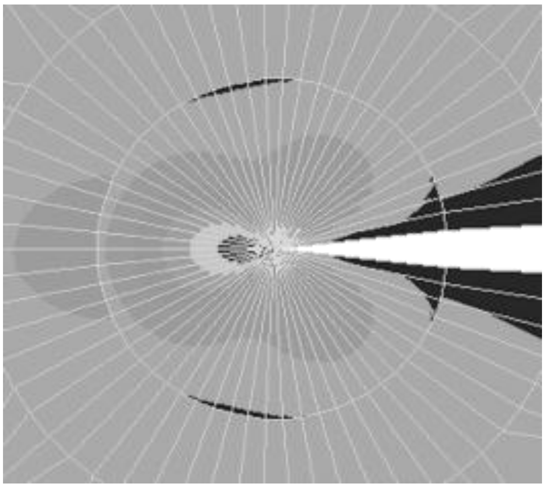

Fig. 7. stress distribution (σ_x) around the crack tip

K_I = 4.8400×10^7 for *Plane stress*, and K_I = 4.4044×10^7 for *Plane strain*.
Error percentage between K_I (calculated in Eqs. (2) – (3)) and K_I (directly obtained from ANSYS software), is 0.6% (less than 1%). This percentage shows that the software answers is too much closed to analytical solutions.

Although Error percentage between K_I (calculated in Eqs. (2) – (3)) and K_I (calculated in Eq. (1)), is 92%, but anyway it shows an answer near to analytical solutions when there is no any parameters available to obtain accurate results.

Cracked beam of Fig. 1 with different supporting conditions and loads is considered and because of the low errors of finite element software (FES), K_I and stress distribution results are obtained from FES. These problems are explained below:

Problem.1: the previous problem that were discussed in Fig. 1.

Problem.2: This problem is like Problem 1, except that it has two points that are fixed and pressure is applied only on upper surface of the beam as shown in Fig. 8.

Problem.3: This problem is like Problem 2 except that the pressure direction is changed and there is no contact defined on crack surfaces as shown in Fig. 9.

Problem.4: This problem is like Problem 3 except that there is contact between crack surfaces as shown in Fig. 10.

Problem.5: This problem is like Problem 3 except that one end of the beam is fixed as shown in Fig. 11.

Problem.6: This problem is like Problem 5 except that there is contact between crack surfaces as shown in Fig. 12.

For these problems, stress intensity factors in *plane strain* and *plane stress* conditions are calculated. Then *normal* and *shear* stress diagrams of some cross section of

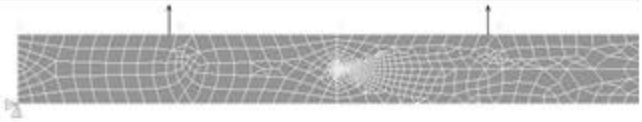

Fig. 8. Problem 2

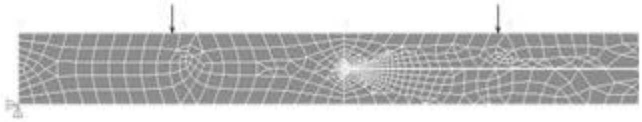

Fig. 9. Problem 3

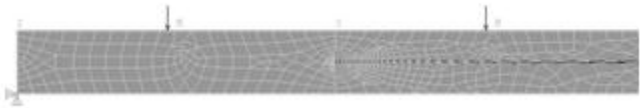

Fig. 10. Problem 4

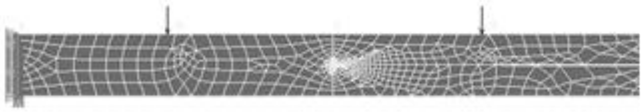

Fig. 11. Problem 5

Fig. 8. Problem 2

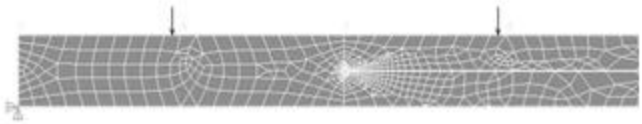

Fig. 9. Problem 3

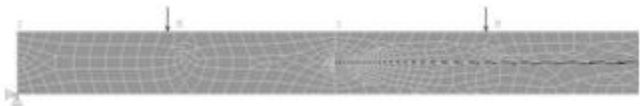

Fig. 10. Problem 4

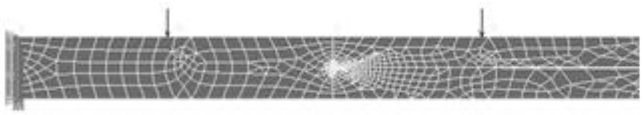

Fig. 11. Problem 5

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the beam are obtained for analyzing stress distribution around the crack tip.

RESULTS AND DISCUSSION

Stress intensity factors are shown in Table 1. Figs 13-17 shown normal and shear stress distributions diagram on cross section of the beam, which passed through the crack tip.

Results in Table 1, show that for all problems, in plane stress, stress intensity factors are 10% less than stress intensity factors in *plane strain*.

Comparison Problems 3 and 4, shows that when there is *no contact* and *frictionless contact* between crack surfaces, difference between stress intensity factors are less than 0.1% when other conditions are the same. This comparison is also true for Problems 5 and 6.

Stress intensity factors in Problems 2 and 3 are the same that shows that if there is no any contact between crack surfaces, there is no matter to apply pressure upward or downward on beam surfaces.

Results show that when there are two-fixed points in the beam, K_I has the most value than other conditions.

Figs 13-17 are shown normal (σ_x) and shear (τ_{xy}) stresses on cross section of the beam. Horizontal axis is shown the distance from one side of beam height to another one. Crack tip is located in the middle of this distance. Vertical axis is shown the stress values.

Figs 13-17 (a) are shown normal stresses. The normal stress diagrams in Figs 13, 15 and 17 are act symmetric whereas in Figs 14 and 16 are not.

When pressure is upward, maximum stress is positive (Fig. 13) and when pressure is downward Maximum stress is negative (Figs 14-17 (a)).

When there is a contact between surfaces (Figs 15 and 17 (a)), maximum stresses are more than no contact conditions (Figs 14 and 16 (a)).

Figs 13-17 (b) are shown shear stresses. In the middle of horizontal axis (crack tip), shear stresses are zero.

When there is a contact between surfaces (Figs 15 and 17 (b)), maximum shear stresses are more than no contact conditions (Fig. 14 and 16 (b)).

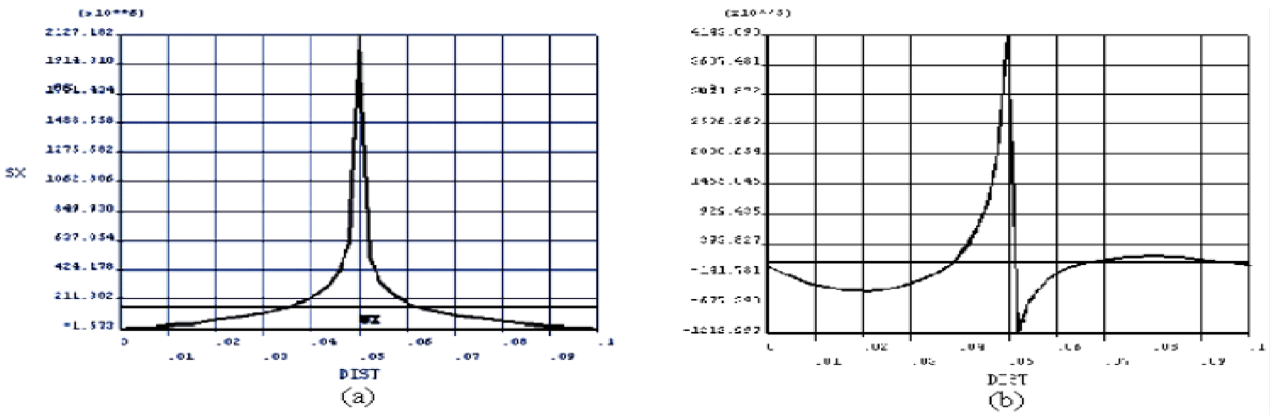

Fig. 13. (a) Normal stress distributions, (b) shear stress distributions for problem 2

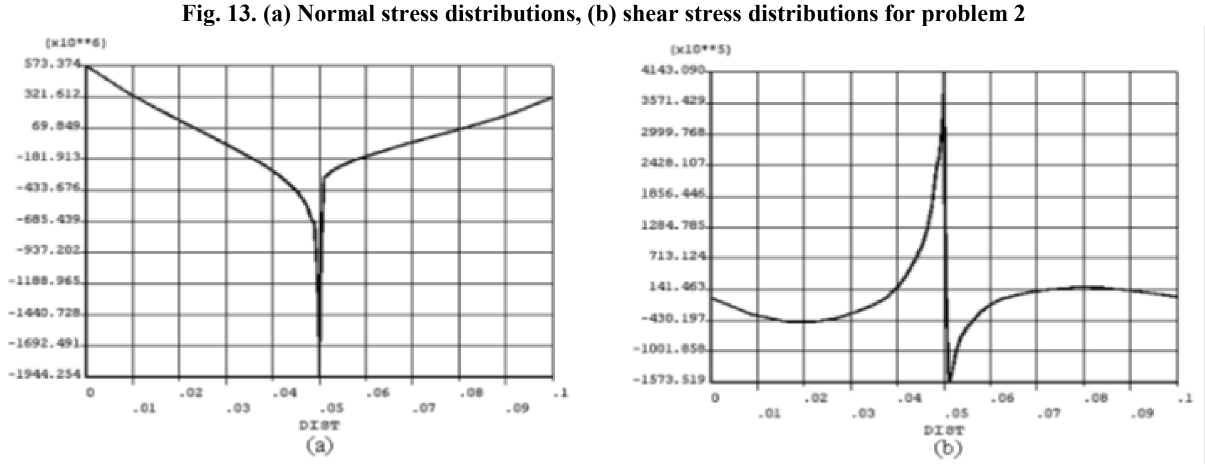

Fig. 14. (a) Normal stress distributions, (b) shear stress distributions for problem 3

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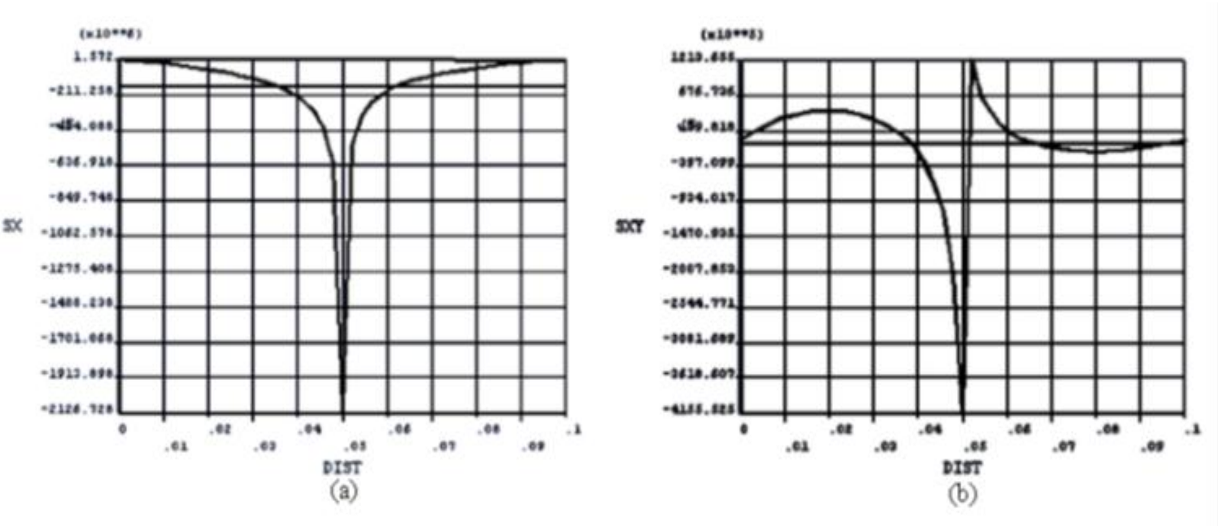

Fig. 15. (a) Normal stress distributions, (b) shear stress distributions for problem 4

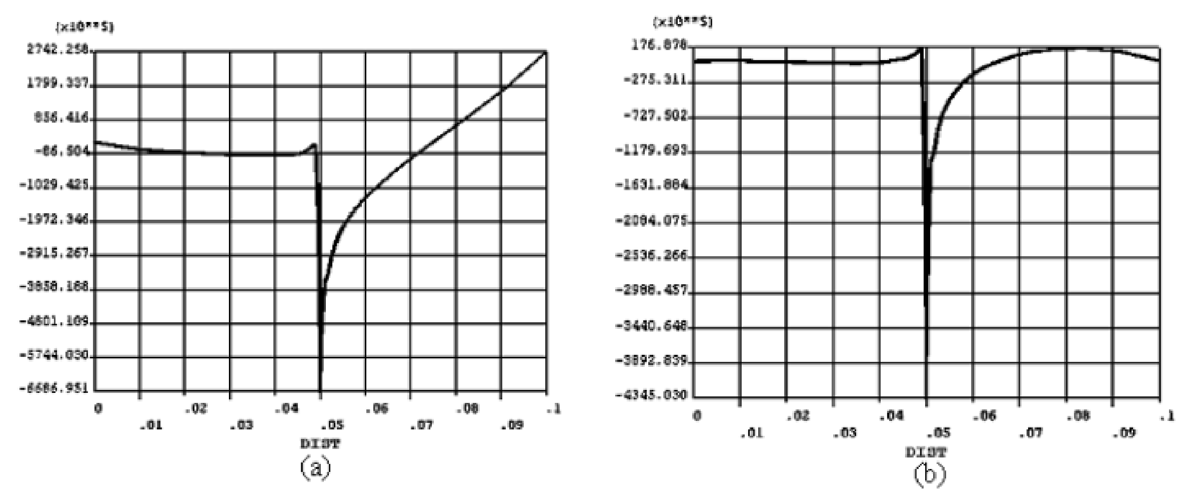

Fig. 16. (a) Normal stress distributions, (b) shear stress distributions for problem 5

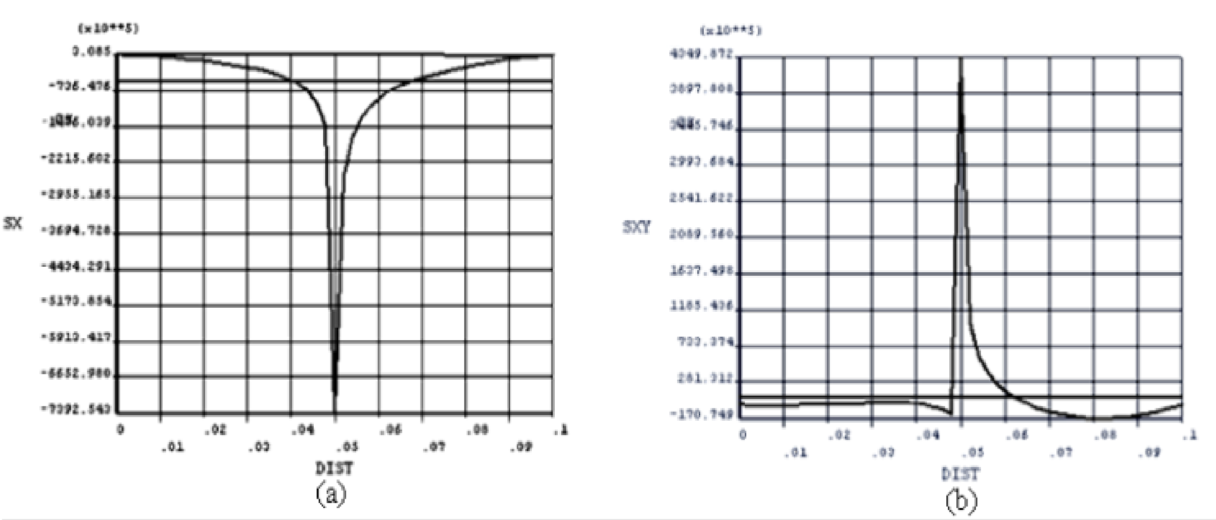

Fig. 17. (a) Normal stress distributions, (b) shear stress distributions for problem 6

CONCLUSION

Results are shown that frictionless contact does not affect beam behavior in prescribed conditions. Comparison of the results shown that stress intensity factors can be implemented for predicting the maximum stresses behavior of the beam that is important for researchers and engineers. Another interesting result is the symmetric behavior of normal stresses diagram around the crack tip when there is a contact between surfaces.

Table 1: Stress Intensity Factors for Different Problems

Kind of problem	K_I					
	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6
Plane stress	4.4044×10^7	6.3405×10^7	6.3405×10^7	6.3419×10^7	2.2047×10^7	2.2028×10^7
Plane strain	4.8400×10^7	6.9676×10^7	6.9676×10^7	6.9691×10^7	2.4228×10^7	2.4207×10^7

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