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Modeling of Swivel Joint in two Dimensional Beams Framework

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ABSTRACT

Generally swivel joints often frameworks, notably beams one, and allow them to swivel in a point around the three space dimensions. In the modeling of those frameworks, swivel joint is supposed to be in a node. Between two elements at a node where there is a swivel joint, displacements are continuous and rotations are not. This paper is a contribution to the calculation of the rotations in a swivel point. Our calculation method is based on the finite elements method and the disturbance of the elementary stiffness matrix through the static condensation. At the end, we present analytical results obtained on the connected elements by moving the swivel joint from one element to another and the proposition to put simultaneously two swivel joint in two related elements.

Keywords*: Beams, Swivel Joint, Finite Elements Method, Static Condensation.*

1. INTRODUCTION

In the finite elements method, the calculation of the degree of freedom of a framework passes through the assembly of all elementary stiffness matrices of the framework [1]. Taking into account of the swivel joint on a beam in the elementary stiffness matrix requires the disturbance of this matrix according to the place of the swivel joint [2]. In the first part of this work we suggest a modeling of the swivel joint in a beam element by disturbance of the elementary stiffness matrix. Afterwards, we propose a calculation method of the rotation on a swivel joint which allows obtaining the same value of the rotation indiscriminately of its position in the two related elements. We also present the technical disturbance when we have swivel-swivel joint in two related elements. The final part of this work is booked to the discussion of our results.

2. STIFFNESS MATRIX OF THE TWO DIMENSIONAL BEAMS FRAMEWORK

With the finite elements method, by taking two nodes elements, we can divide the beams framework in two elements of figure 1. The first element have the nodes (1,2), and the second elements have the node (2,3):

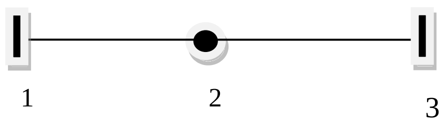

Figure 1: Plan of a beams framework made with two elements

If we suppose that each element has two degrees of freedom, the displacement following the (oy) axis: u, and the rotation around the (oz) axis: θ. As shown in figure 2, there are four degrees of freedom for one element.

Figure 2: Degrees of freedom on a beam element

If we consider that at each node, there is elementary load with two components: $F_i\begin{pmatrix} f_i \\ m_i \end{pmatrix}$, f_i : the component load following the (oy) axis, m_i : the component load around the (oz) axis. The elementary system of the equilibrium equations of the element (1) can be formulated as follows:

$$\begin{bmatrix} K^1_{u_1u_1} & K^1_{u_1\theta_1} & K^1_{u_1u_2} & K^1_{u_1\theta_2} \\ K^1_{\theta_1u_1} & K^1_{\theta_1u_2} & K^1_{\theta_1u_2} & K^1_{\theta_1\theta_2} \\ K^1_{u_2u_1} & K^1_{u_2\theta_1} & K^1_{u_2u_2} & K^1_{u_2\theta_2} \\ K^1_{\theta_2u_1} & K^1_{\theta_2\theta_1} & K^1_{\theta_2u_2} & K^1_{\theta_2\theta_2} \end{bmatrix} \begin{pmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ m_1 \\ f_2 \\ m_2 \end{pmatrix} \quad (1)$$

The elementary system of the equilibrium equations of element (2) is of the same kind. By assuming that node (2) is set in, the assembly of the elementary stiffness matrices and the elementary load vector with the limit conditions, the global final equations system obtained is:

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$$\begin{bmatrix}\left(K^1_{u_2u_2}+K^2_{u_2u_2}\right) & \left(K^1_{u_2\theta_2}+K^2_{u_2\theta_2}\right)\\ \left(K^1_{\theta_2u_2}+K^2_{\theta_2u_2}\right) & \left(K^1_{\theta_2\theta_2}+K^2_{\theta_2\theta_2}\right)\end{bmatrix}\begin{pmatrix}u_2\\ \theta_2\end{pmatrix}=\begin{pmatrix}f_2^1+f_2^2\\ m_2^1+m_2^2\end{pmatrix} \qquad (2)$$

In this system, where components K^i and f^i are related to (i) element, we can determine the upright displacement (u_2) and the rotation around (θ_2) of the set node (2). When there is swivel at the node (2), the calculations are different.

3. TAKING INTO ACCOUNT OF SWIVEL JOINTS

Assuming that we have a swivel joint in the node (2), as represented in the figure (3), there are two ways to modeling this joint.

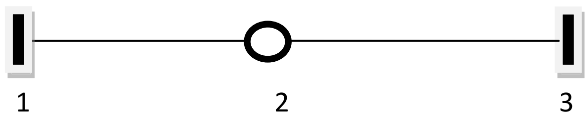

Figure 3: Beams framework with a swivel joint in node (2)

The swivel joint can either be on the second node of element (1) or on the first node of element (2).

3.1 Case where the swivel joint is supposed to be in the second node of element (1)

In this case, we have the following figure 4:

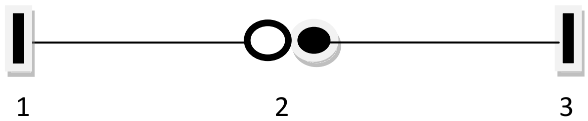

Figure 4: Swivel joint in the second node of element (1)

The rotation value of node (2) is discontinued because of the presence of the swivel joint. At this point, there is a swivel node and a setting in one. The rotation of the swivel joint node is calculated with elementary equations and the rotation of the set in node is calculated by the global equations. Considering the swivel joint on node (2) of the element (1) consists in cancelling the internal force at this node. At this node the internal force which is related to the bending momentum must also be cancelled: $(m_2^1=0)$ [3]. The elementary system equations of this element is disturbed and becomes:

$$\begin{bmatrix}K^1_{u_1u_1} & K^1_{u_1\theta_1} & K^1_{u_1u_2} & K^1_{u_1\theta_2}\\ K^1_{\theta_1u_1} & K^1_{\theta_1u_2} & K^1_{\theta_1u_2} & K^1_{\theta_1\theta_2}\\ K^1_{u_2u_1} & K^1_{u_2\theta_1} & K^1_{u_2u_2} & K^1_{u_2\theta_2}\\ K^1_{\theta_2u_1} & K^1_{\theta_2\theta_1} & K^1_{\theta_2u_2} & K^1_{\theta_2\theta_2}\end{bmatrix}\begin{pmatrix}u_1\\ \theta_1\\ u_2\\ \theta_2\end{pmatrix}=\begin{pmatrix}f_1\\ m_1\\ f_2\\ m_2=0\end{pmatrix} \qquad (4)$$

From static condensation [1], on can determine the local rotation θ^1_{2l} of the swivel rotation of element (1) through the following relationship:

$$\theta^1_{2l}=-\frac{1}{K^1_{\theta_2\theta_2}}\left(K^1_{\theta_2u_1}u_1+K^1_{\theta_2\theta_1}\theta_1+K^1_{\theta_2u_2}u_2\right) \qquad (5)$$

The value θ^1_{2l} is injected in the elementary equations system which becomes disturbed and can be written as:

$$\begin{bmatrix}K^1_{u_1u_1}-\frac{K^1_{u_1\theta_2}K^1_{\theta_2u_1}}{K^1_{\theta_2\theta_2}} & K^1_{u_1\theta_1}-\frac{K^1_{u_1\theta_2}K^1_{\theta_2\theta_1}}{K^1_{\theta_2\theta_2}} & K^1_{u_1u_2}-\frac{K^1_{u_1\theta_2}K^1_{\theta_2u_2}}{K^1_{\theta_2\theta_2}} & 0\\ K^1_{\theta_1u_1}-\frac{K^1_{\theta_1\theta_2}K^1_{\theta_2u_1}}{K^1_{\theta_2\theta_2}} & K^1_{\theta_1\theta_1}-\frac{K^1_{\theta_1\theta_2}K^1_{\theta_2\theta_1}}{K^1_{\theta_2\theta_2}} & K^1_{\theta_1u_2}-\frac{K^1_{\theta_1\theta_2}K^1_{\theta_2u_2}}{K^1_{\theta_2\theta_2}} & 0\\ K^1_{u_2u_1}-\frac{K^1_{u_2\theta_2}K^1_{\theta_2u_1}}{K^1_{\theta_2\theta_2}} & K^1_{u_2\theta_1}-\frac{K^1_{u_2\theta_2}K^1_{\theta_2\theta_1}}{K^1_{\theta_2\theta_2}} & K^1_{u_2u_2}-\frac{K^1_{u_2\theta_2}K^1_{\theta_2u_2}}{K^1_{\theta_2\theta_2}} & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{pmatrix}u_1\\ \theta_1\\ u_2\\ \theta_2\end{pmatrix}=\begin{pmatrix}f_1\\ m_1\\ f_2\\ m_2=0\end{pmatrix} \qquad (6)$$

Thus on can realize that the static condensation don't concern elementary external forces, it only affects the elementary rigidity matrix. In order to calculate displacement and rotation θ^1_{2g} of set node which is global, we must proceed to the assembly of the elementary external loads and the assembly of the elementary stiffness matrices. And we obtain the equations system (7):

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$$\begin{bmatrix} \left(\left(K^1_{u_2u_2} - \frac{K^1_{u_2\theta_2}K^1_{\theta_2u_2}}{K^1_{\theta_2\theta_2}}\right) + K^2_{u_2u_2}\right) & K^2_{u_2u_2} \\ K^2_{\theta_2u_2} & K^2_{\theta_2\theta_2}\end{bmatrix}\begin{pmatrix}u_2\\ \theta^1_{2g}\end{pmatrix} = \begin{pmatrix} f^1_2+f^2_2 \\ 0+m^2_2\end{pmatrix} \quad (7)$$

3.2 Case where the swivel joint is on the element (2)

This case is illustrated by figure 5. We have the plan:

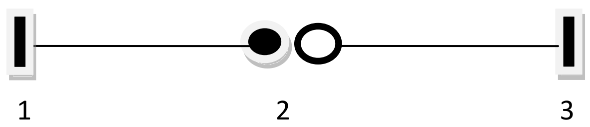

Figure 5: Case where the swivel joint is supposed to be in the first node on the second element

Putting swivel joint on the local node (1) of element (2) amounts to cancel the internal force that is, the bending momentum at this node. In the similar way as we did before by using this time the local node (1) of element (2), but which is the global node (2) of our framework. We obtain equations system (8):

$$\begin{bmatrix} K^2_{u_2u_2} & K^2_{u_2\theta_2} & K^2_{u_2u_3} & K^2_{u_2\theta_3} \\ K^2_{\theta_2u_2} & K^2_{\theta_2\theta_2} & K^2_{\theta_2u_3} & K^2_{\theta_2\theta_3} \\ K^2_{u_3u_2} & K^2_{u_3\theta_2} & K^2_{u_3u_3} & K^2_{u_3\theta_3} \\ K^2_{\theta_3u_2} & K^2_{\theta_3\theta_2} & K^2_{\theta_3u_3} & K^2_{\theta_3\theta_3} \end{bmatrix}\begin{pmatrix}u_2\\ \theta_2 \\ u_3 \\ \theta_3\end{pmatrix} = \begin{pmatrix} f_2 \\ m_2 \\ f_3 \\ m_3\end{pmatrix} \quad (8)$$

Through static condensation, we calculate θ^2_{2l} which is the rotation of the swivel joint node. And we obtain the value:

$$\theta^2_{2l} = -\frac{1}{K^2_{\theta_2\theta_2}}\left(K^2_{\theta_2u_2}u_2 + K^2_{\theta_2u_3}u_3 + K^2_{\theta_2\theta_3}\theta_3\right) \quad (9)$$

When we put the algebraic expression of θ^2_{2l} in the system (8), and we make the assembly of the elementary external forces and stiffness matrix, we obtain the following global equations system:

$$\begin{bmatrix} \left(K^1_{u_2u_2} + \left(K^2_{u_2u_2} - \frac{K^2_{u_2\theta_2}K^2_{u_2\theta_2}}{K^2_{\theta_2\theta_2}}\right)\right) & K^1_{u_2\theta_2} \\ K^1_{\theta_2u_2} & K^1_{\theta_2\theta_2}\end{bmatrix}\begin{pmatrix}u_2\\ \theta^2_{2g}\end{pmatrix} = \begin{pmatrix} f^1_2+f^1_2 \\ m^1_2+0\end{pmatrix} \quad (10)$$

This system evaluates the expression of θ^2_{2g} which is the rotation of the set node at this point.

3.3 Case where the swivel joint is jointly on the two elements

In that case, we have the following plan:

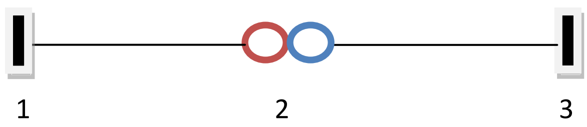

Figure 6: Case where the swivel joint is simultaneously on two related elements

In the case, we suggest to disturb simultaneously the elementary rigidity matrices of the two elements. In additions, as in the case where the global node (2) was supposed to set in the two related elements, we suggest its rotation to be continued between the two elements. In order to solve this problem, we suggest to calculate this unique rotation value after disturbance of both elementary matrices rigidity and its assembly.

Our suggestion during disturbance concerns conservation of values of the line and to have zero values in the node where there is a swivel joint in order to evaluate the algebraic expression on the mutual rotation. Hence we obtain the following matrix by eliminating the matrix elements of the fourth column except the element of the swivel node (2) of the element (1). We use the same method as previously, but this time for the second column and the element stiffness matrix of the node (1) of element (2). After assembly, we obtain the following system equations:

$$\begin{bmatrix} \left(K^1_{u_2u_2} + \left(K^2_{u_2u_2} - \frac{K^2_{u_2\theta_2}K^2_{u_2\theta_2}}{K^2_{\theta_2\theta_2}}\right)\right) & K^1_{u_2\theta_2} \\ \left(K^1_{\theta_2u_2} + K^2_{\theta_2u_2}\right) & \left(K^1_{\theta_2\theta_2} + K^2_{\theta_2\theta_2}\right)\end{bmatrix}\begin{pmatrix}u_2\\ \theta_{2ll}\end{pmatrix} = \begin{pmatrix} f^1_2+f^2_2 \\ 0+0\end{pmatrix} \quad (13)$$

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The value θ_{2ll} of the global node (2), where we have two swivel joints, is obtain after resolution of the preceding system:

$$\theta_{2ll} = -\frac{\left(K^1_{\theta_2 u_2} + K^2_{\theta_2 u_2}\right)}{\left(K^1_{\theta_2\theta_2} + K^2_{\theta_2\theta_2}\right)} u_2 \qquad (14)$$

According to our modeling, three beams are related by a swivel joint as it is shown on figure 7:

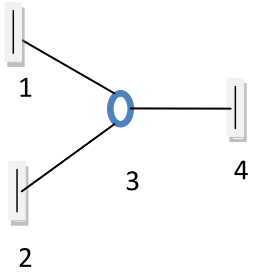

Figure 7: Beams framework with three elements related by a swivel joint

This framework could only be modelling this by the following plans (figure 8: a, b, c):

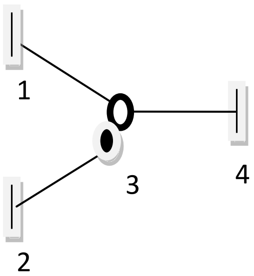

Figure 8.a

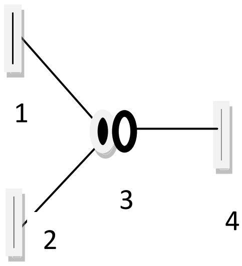

Figure 8.b

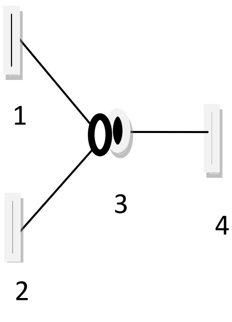

Figure 8.c

Figure 8: Different ways of modeling a swivel joint between three beams

4. ANALYTICAL RESULTS AND DISCUSSION

We take again the framework shown in figure 1. By suggesting that, beams are identical and that, only element (2) is charged, with nodes (1) and (3) set in as border conditions: F_1=0. Hence we have:

$$F_1 \begin{pmatrix} f_1^1 = 0 \\ m_1^1 = 0 \\ f_2^1 = 0 \\ m_2^1 = 0 \end{pmatrix} \qquad (15)$$

For F_2, we have:

$$F_2 \begin{pmatrix} f_2^2 \\ m_2^2 = 0 \\ f_3^2 \\ m_3^2 = 0 \end{pmatrix}. \qquad (16)$$

We obtain the following results:

i. in the case where the swivel joint is on the node (2) of element (1):

The rotation of the swivel node is:

$$\theta^1_{2l} = -\frac{K^1_{\theta_2 u_2}}{K^1_{\theta_2\theta_2}} u_2 \qquad (17)$$

and the rotation of setting node is:

$$\theta^1_{2g} = -\frac{K^2_{\theta_2 u_2}}{K^2_{\theta_2\theta_2}} u_2 \qquad (18)$$

ii. In the case where the swivel joint is on the node (1) of element (2), we have :

The rotation of the swivel node is:

$$\theta^2_{2l} = -\frac{K^2_{\theta_2 u_2}}{K^2_{\theta_2\theta_2}} u_2 \qquad (19)$$

And the rotation of setting node is:

$$\theta^2_{2g} = -\frac{K^1_{\theta_2 u_2}}{K^1_{\theta_2\theta_2}} u_2 \qquad (20)$$

iii. For the case of double swivel joint, we have the following value of the rotation at this node:

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$$\theta_{2ll} = -\frac{\left(K^{1}_{\theta_2 u_2} + K^{2}_{\theta_2 u_2}\right)}{\left(K^{1}_{\theta_2\theta_2} + K^{2}_{\theta_2\theta_2}\right)} u_2 \qquad (21)$$

If the beams of the framework are identical, it comes that: $\theta^1_{2l} = \theta^2_{2l}$ and $\theta^1_{2g} = \theta^2_{2g}$. More over, we obtain in the case of double swivel joint case: $\theta_{2ll} = \frac{1}{2}\left(\theta^1_{2l} + \theta^2_{2l}\right)$. Like in the case where node (2) is setting in both related beams, we have the rotations in continue. Also we have established that, when the node is swivel jointed in both related beams, rotation value is also continue.

5. SUMMARY

In this contribution, through technical matrices based on the finite element method, we have calculated rotations values around the swivel joint. We have shown that, no matter the swivel joint is supposed to be in two related beams in a two dimensional framework, we obtain the same value. Moreover, we have suggested a modeling of putting a swivel joint in two related beams which permits to have a unique rotation value in node, as we have a unique rotation value in a joint setting node.

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