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An Unsteady Forced and Free Convection Flow Past an Infinite Permeable Vertical Plate

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ABSTRACT

A suction/injection (s) controlled free convection motion of a viscous incompressible fluid past an infinite permeable vertical plate considered. The expression for temperature (θ) and velocity (u) fields obtained and using laplace transform technique. The influence of each variable parameter is discussed with the aid of graphs. It is found that temperature (θ) is higher near the plate with injection while velocity (u) is more enhanced near the plate with both suction and injection. Also it is observed that the skin friction(τ) decreases with increasing suction/injection (s) when time (t) is constant.

Keywords: *Suction/Injection, Free convection, Prandtl number.*

1. INTRODUCTION

The study on the unsteady free convection flow under different conditions past an infinite vertical plate where done by Siegel [1958], Schetz and Eichhorn [1962], Menold and Yang [1962], Chung and Anderson [1962], Goldstein and Briggs [1964] and Sugawava and Michiyoshi [1951].Goldstein and Eckert [1960] confirmed experimentally some of these theoretical predictions. In all these studies, the infinite plate was assumed to be stationary and the fluid was supposed to move due to temperature difference only.

If the fluid is stationary and the infinite plate surrounded by the stationary fluid is given, an impulsive motion along with its temperature raised to T_w such that $T'_w > T_\infty$, where T_∞ is the temperature of the surrounding fluid. How the flow of the fluid takes its shape? This was studied by Soundalgakar [1960] in the case of an isothermal plate. The effect of free convection current on the flow and the skin-friction was also studied by Jahagirdar and Lahurikar [1989].

Suction or injection of a fluid through the boundary surface, like for example in mass transfer cooling, can significantly change the flow flied and, as a consequence, after the heat transfer rate of the plate (Jha and Ajibade[2010] and Ishak,Merkin,Nazar and Pop[2008]). In general, suction tend to increase the skin friction and heat transfer coefficient where as injection acts in the opposite manner (Al-Sanea [2004]). Suction and injection plays an important role in the control of flow past an infinite plate, hence it's importance in practical problems involving film cooling, control of boundary layers in industrial, geophysical, biomedical, engineering and environmental applications.

Shojaefard et al. [2005] investigated flow control on a subsonic airfoil by suction and injection and concluded that suction significantly increases the lift coefficient while injection decreases the surface skin friction, which transitively resulted in a considerable reduction in energy consumed during flights of subsonic aircraft.

The aim of this paper is to extend the work of Jahagirdar and Lahurikar [1989] to the case when plates are permeable, i.e. when there is suction or injection through the plates bounding the fluid. Fully developed free convection flows are considered in this problem because the study of such flow gives the limiting conditions for developing flows and provides an analytical check on numerical solution. Exact solutions are obtained in the present work, which are very important as they serve as accuracy checks for experimental and asymptotic method.

2. MATHEMATICAL ANALYSIS

A natural unsteady free and forced convection flow of a viscous incompressible fluid past an infinite heated vertical permeable plate is considered with suction and injection. The permeable plate is taken vertically, parallel to the x'-axis and the y'-axis is taken normal to the plate (see Fig.1). Then the physical variables are functions of y' and t' only.

We consider a two dimensional flow so that $\vec{V} = (u, v, 0)$, where u and v are the vertical and horizontal components of velocity, respectively. The flow is along the x'-axis which is a full flow, and is a function of y' only. The equation of continuity yields

$$\frac{du}{dy} = 0 \qquad (1)$$

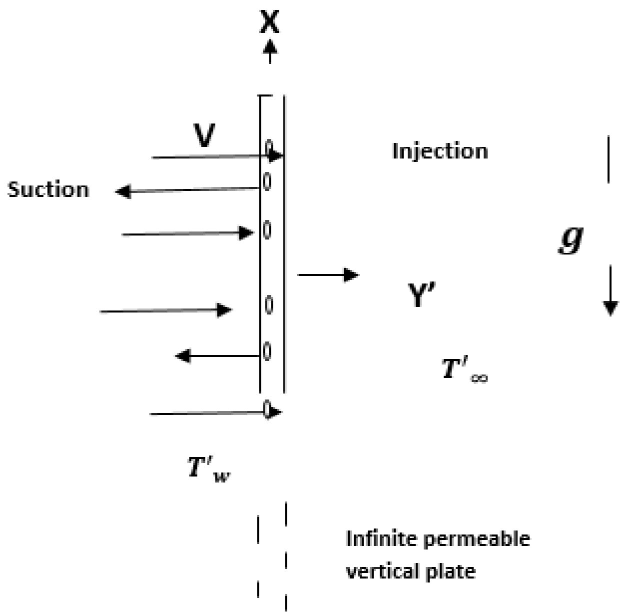

Fig. 1 Schematic Diagram of the Problem

Integrating Eq.(1) we have the horizontal velocity as $u = v_0$ (a constant) which is the velocity of suction/injection.

Using the usual Boussinesq's approximation, the momentum and energy equations yields

$$\rho\frac{\partial u'}{\partial t'} - V_0\rho\frac{\partial u'}{\partial y'} = g\beta\rho(T' - T'_\infty) + \mu\frac{\partial^2 u'}{\partial y'^2} \quad (2)$$

$$\rho C_p\frac{\partial T'}{\partial t'} - V_0\frac{\partial T'}{\partial y'} = k\frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

The initial and boundary conditions are

$$t' \le 0, u' = 0, T' \to T'_\infty, \text{For all } y'$$
$$t' > 0, u' = 0, T \to T'_w \; at \; y' = 0 \quad (4)$$
$$u' \to u_o, T' \to T'_\infty \; as \; T' \to \infty.$$

On introducing the non-dimensional quantities

$$y = \frac{y'u_0Gr^{1/2}}{v}, t = \frac{t'u_0^2Gr}{v}, u = \frac{u'}{u_0}, Pr = \frac{\mu C_p}{k}, \theta = \frac{(T'-T'_\infty)}{(T'_w-T'_\infty)} \; and \; Gr = \frac{vg\beta(T'_w-T'_\infty)}{u_0^3} \; [the\ Grashof\ number]. \quad (5)$$

Substituting Eq.(5) in Eqs.(2) and (3) and applying boundary condition (4), we obtained

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = \theta + \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

Subject to the following boundary conditions

$$t \le 0, u = 0, \theta = 0, for\ all\ y$$
$$t > 0, u = 0, \theta = 1, at\ y = 0$$
$$u = 1, \theta = 0, as\ y \to \infty \quad (8)$$

Where

$$S = \frac{V_0}{u_0}\frac{1}{Gr^{1/2}}, Pr = 1$$

Pr is the prandtl number, and S is the suction/injection parameter.

Solving Eqs (6) and (7) with the boundary conditions (8) by the laplace transform technique, we obtained

$$\theta = \frac{\exp(-z)}{2}\{\exp(\psi)\, erfc(\eta + \varphi) + \exp(-\psi)\, erfc(\eta - \varphi)\}. \quad (10)$$

$$u = 1 - \frac{\exp(-z)}{2}\{\exp(\psi)\, erfc(\eta + \varphi) + \exp(-\psi)\, erfc(\eta - \varphi)\} + \frac{y exp(-\propto y)\exp(-\propto^2 t)}{2}\{\frac{2}{s^2}[\exp(\propto^2 t - \propto y)\, erfc(\eta - \propto\sqrt{t}) - \exp(\propto^2 t + \propto y)\, erfc(\eta + \propto\sqrt{t})]\}$$

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(11)

We now study the skin- friction. It is given by

$$\tau' = -\mu \frac{\partial u'}{\partial y'} \quad at\ y = 0 \qquad (12)$$

And in view of Eq.(5), Eq.(12) reduces to

$$\tau = -\frac{\tau'}{\rho u_0^2 Gr^{1/2}} = \frac{\partial u}{\partial y}|_{y=0} \qquad (13)$$

Finding $\frac{\partial u}{\partial y}$ at $y = 0$ from Eq.(11) and substituting in eq.(13) and simplifying, we get the required skin-friction as;

$$\tau = \frac{\exp(-s^2 t)}{\sqrt{\pi t}} + \frac{s}{4}\{3erfc(-s\sqrt{t}) - erfc(s\sqrt{t})\} - \frac{1}{2s}\{ erfc\left(-\frac{s}{2}\sqrt{t}\right) + erfc\left(\frac{s}{2}\sqrt{t}\right) \qquad (14)$$

3. RESULTS AND DISCUSSION

Three basic parameters governed the flow through an infinite porous vertical plate namely the prandtl number (Pr), which is inversely proportional to the thermal diffusivity of the working fluid, the distance the fluid moves (y) on the y' direction, and suction and injection parameter (s), which where applied to the infinite plate.

Contour graphs of the induced unsteady temperature and velocity profile as well as the skin friction (τ) profile are presented in figures 1-10, to reveal the influence of the variable parameter on the temperature, velocity and the skin friction(τ).

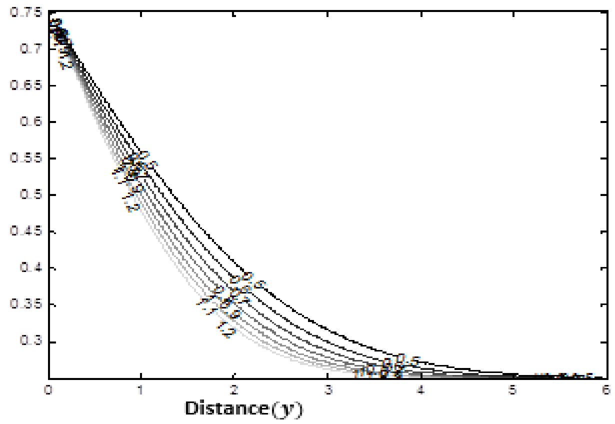

Fig.1 Temperature profile (θ) for different t $(s = 0.0)$

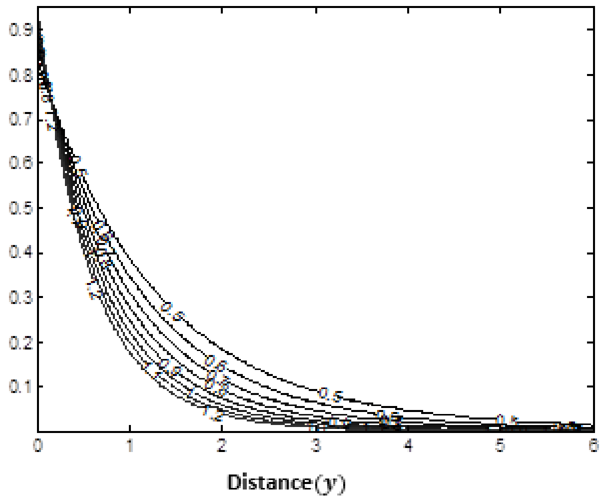

Fig.2 Temperature profile(θ) for different $t(s = 0.5)$

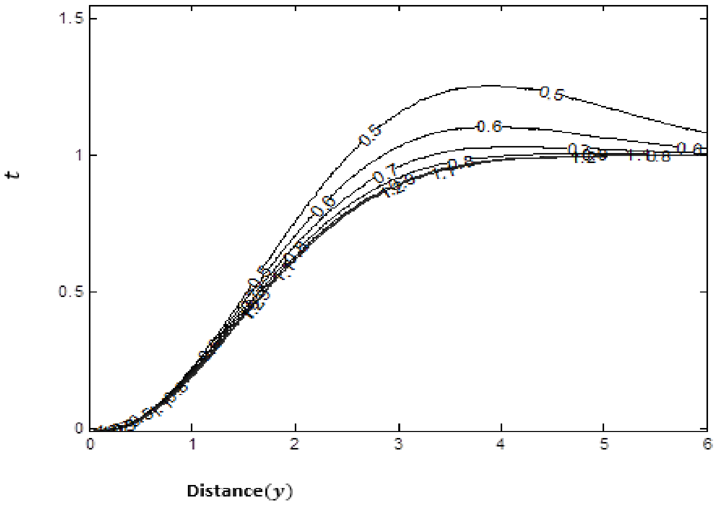

Fig.3 Temperature profile (θ) for different t $(s = -0.5)$

Figures 1-3 shows the variation of temperature for different values of suction/injection (s) as the time (t) increases.

In figure 1, temperature is observed to decrease as the time (t) increases, with the fixed value of s = 0.0. A sharp decay is noticed in temperature near the plate, In Figure 2, Temperature is observed to decrease as time (t) increases, with the constant value of s = 0.5. A sharp decay is also noticed in temperature throughout the plate. While in Figure 3, temperature is observed to increase as time (t) increases, with the constant value of s = -0.5.

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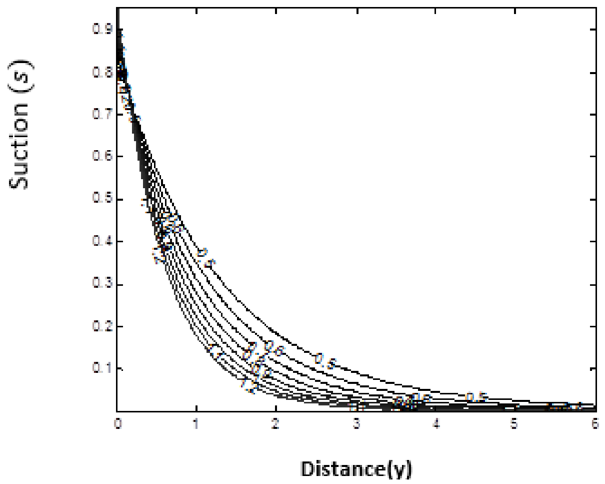

Fig.4 Temperature profile (θ) for different suction/injection (s) ($t = 0.1$)

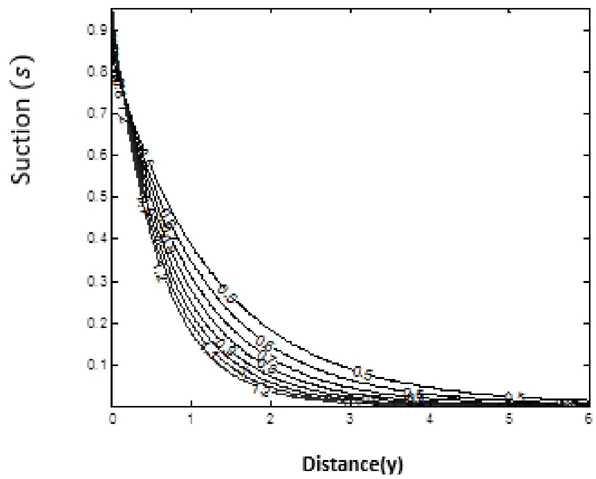

Fig.5 Temperature profile (θ) for different suction/injection (s) ($t = 0.4$)

Figures 4 and 5, shows the variation of temperature for different values of time (t) as suction/injection (s) increases.

In figures 4 and 5, temperature is observed to also decrease as the suction/injection (s) increases, when the time (t) takes the values of $t = 0.1$ *and* $t = 0.4$ .In addition, a sharp decay is noticed in temperature near the plate in both graphs.

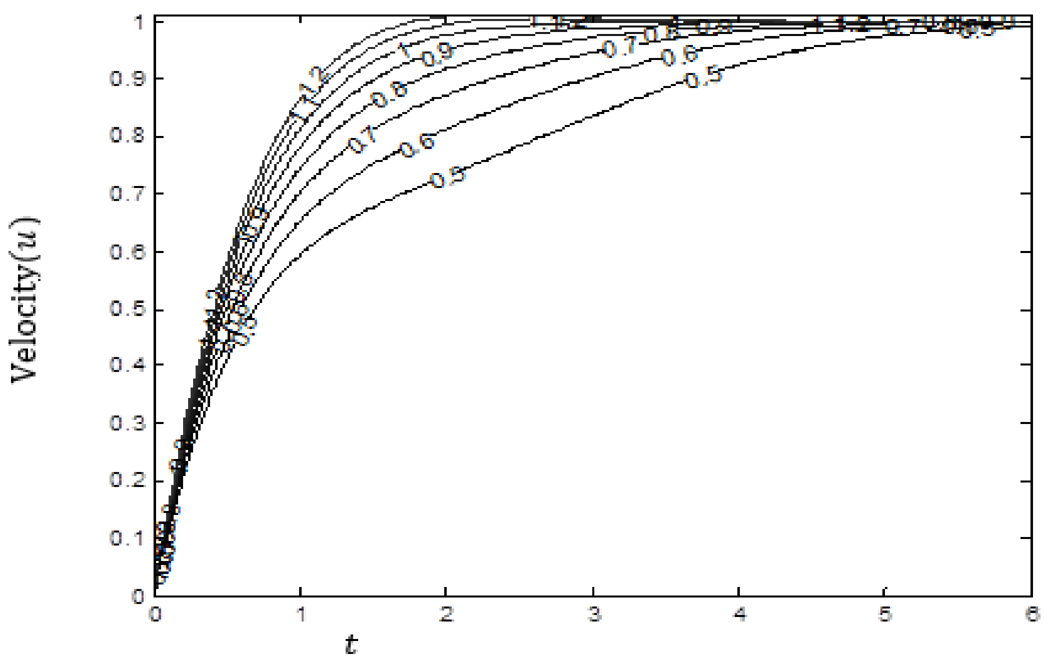

Fig.6 velocity profile (u) for different t ($s = 0.5$)

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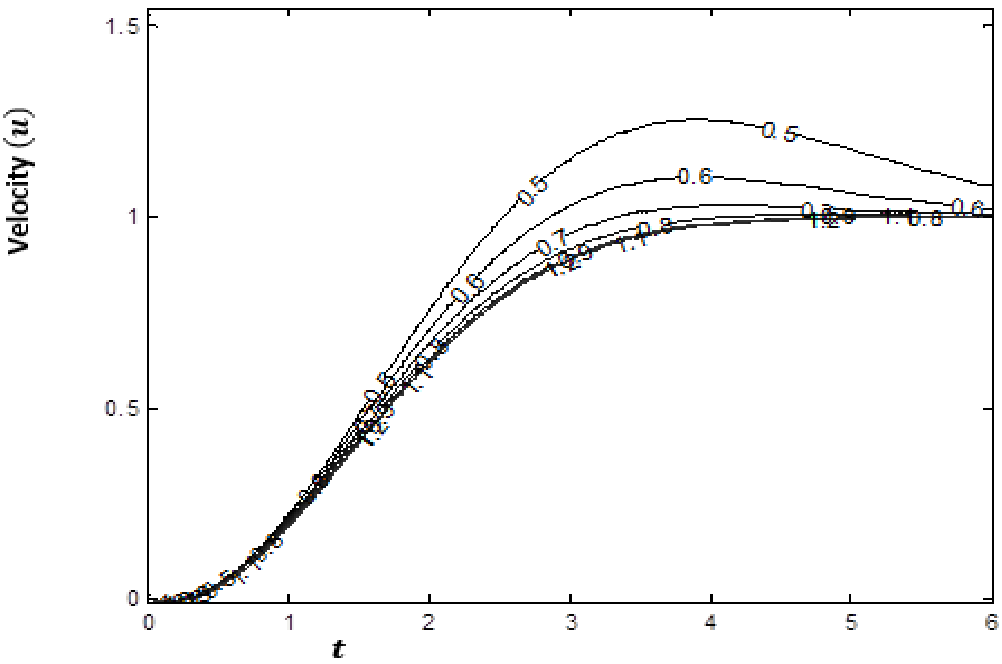

Fig.7 velocity profile (u) for different t $(s = -0.5)$

Figures 6 and 7, shows the variation of velocity (u) for different values of suction/injection (s) as the time t increases.

In figure 6, the velocity (u) is observed to increase as time increases, When $s = 0.5$. Here a sharp decay is observed near the plate as it converges to a point far from the origin. And in figure 7, the velocity (u) is also observed to increase as time increases, When $s = -0.5$ .But in figure 6, a sharp decay is observed not close to the plate but a little further away from the plate and also tend to converge to a point as t increases.

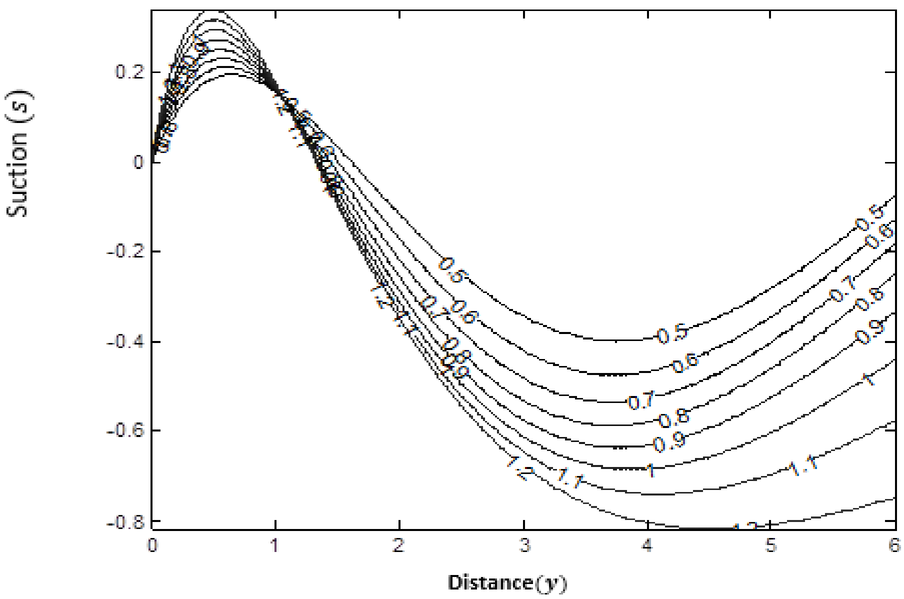

Fig.8 velocity profile (u) for different suction/injection (s) $(t = 0.1)$

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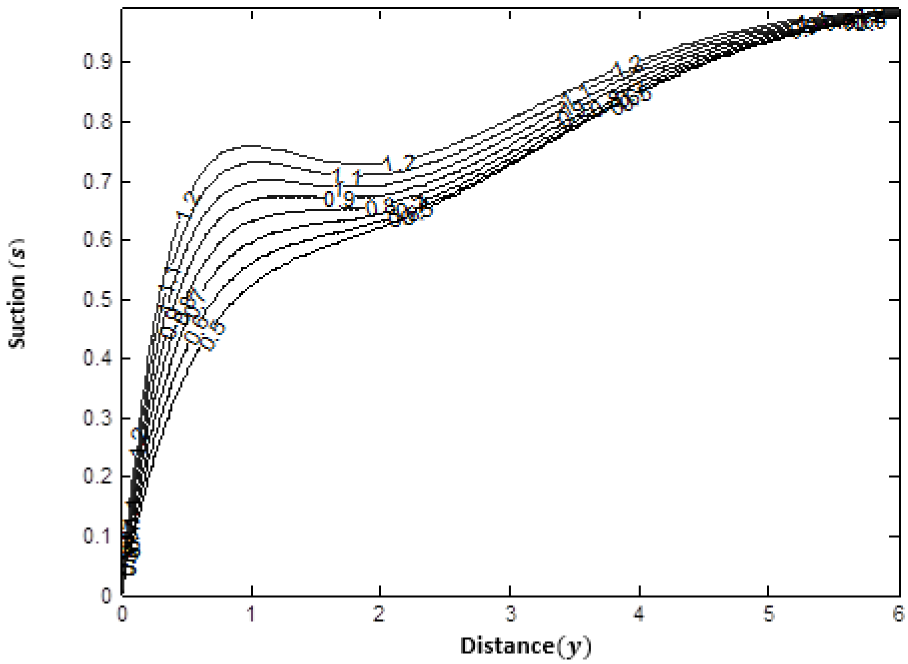

Fig.9 velocity profile (u) for different suction/injection (s)

Figures 8 and 9, shows the variations of velocity (u) for different values of time as suction/injection (s) increases.

In figure 8, the velocity (u) is observed to decrease as suction/injection (s) increases, when $t = 0.1$. A sharp decay near the plate is noticed and at a point when suction/injection (s) is 1, all the points coincide, Showing that the fluid is stationary position at that point.

While in figure 9, the velocity (u) is observed to increase as suction/injection (s) increases, when $t = 0.4$. Also a sharp decay near the plate is noticed and at a distance far from the plate.

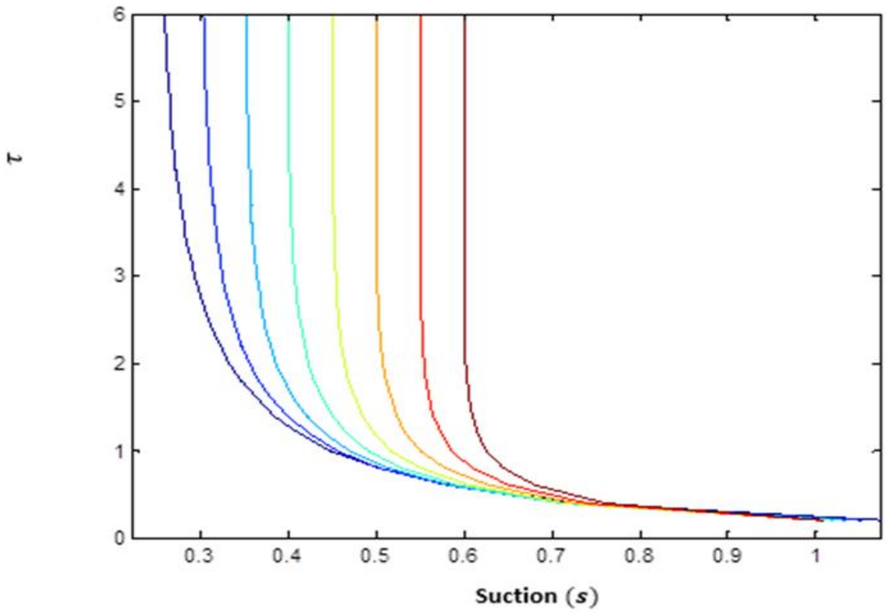

Fig.10 Skin friction (τ) for different suction/injection (s)

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Figure 10, shows the skin friction (τ) profile, where the skin friction (τ) is observed to decrease to a point as suction/injection (s) increases. From what is seen on the graph, it is noticed that there isa sharp decay at points not close to the plate.

4. CONCLUSION

The effects of suction/injection (s), the Prandtl number (Pr) on natural convection flow through an infinite permeable vertical plate is investigated. The expression for velocity (u), temperature (θ) and the skin friction (τ) where obtained. The introduction of suction/injection (s) has changed the nature of the flow by Jahagirdar M.D and Lahurikar R.M [1989].

It is observed that thermal boundary layers increased towards the plate with injection and reduced towards the plate with suction. It is also seen that the temperature (θ) is higher near the plate with injection, while velocity (u) is enhanced near the plate with suction and injection [fig.6 and 7]. The skin friction (τ) decreases with increased in suction/injection (s) when time (t) is constant.

5. NOMENCLATURE

- U' The velocity of the fluid in the x'-direction
- ρ' Density of the fluid
- g Acceleration due to gravity
- β Coefficient of volume expansion
- T' Temperature of the fluid near the plate [Initial]
- T'_∞ Temperature of the fluid in the free stream [Initial]
- μ Coefficient of viscosity
- C_p Specific heat at constant pressure
- K Thermal conductivity
- T'_w Final plate temperature
- Gr Grashof number
- Pr Prandtl number
- t Time, dimensionless parameter defined by equation (3.1.4)
- θ Dimensionless temperature define in equation (3.1.4)
- u Dimensionless velocity of the fluid define in equation (3.1.4)
- y Dimensionless stream wise coordinate define in equation (3.1.4)
- x' Transverse coordinate of the plate in the vertical upward direction
- y' Stream wise coordinate taken normal to the plate
- ν = μ/ρ Kinematic viscosity

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