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The Stability Analysis of the Endemic Equilibrium State of Ebola Virus Disease Transmission in West Africa

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ABSTRACT

The Ebola virus disease is a very infectious and viral disease that is currently ravaging most part of West Africa causing so much mortality and threatening the lives of so many others. If left uncontrolled it could spread to other parts of the world and become a global threat to human existence. In order to help in the global effort to control the spread or even eradicate the disease we present a mathematical model based on the standard SEIR model, in which the population is partitioned into compartments or classes based on the epidemiological state of individuals within the population. The endemic equilibrium point of the model was established and its stability analysis carried out using the basic reproduction number, R_o of Ebola. From the stability analysis it was found out that the Ebola virus disease could be controlled or even be eradicated if effort is made to ensure that the total breakdown of the Susceptible and the latent classes is always less than the total removal rates from the susceptible, the latent and the infectious classes. Some recommendations that would help to meet the established condition were made.

Keywords: *Endemic Equilibrium, Stability Analysis, Reproduction Number, Routh-Hurwitz Criterion*

1. INTRODUCTION

Ebola virus disease is a viral disease caused by four of five viruses classified in the genus Ebola virus. The origin of the disease is somewhat obscure but it was first discovered in Sudan and the Democratic Republic of Congo (then called Zaire) in 1976 when outbreaks occurred. The disease had been endemic in areas in central Africa until the current outbreak in West Africa which began in Guinea in December 2013 and then spread to Liberia, Sierra Leone and Nigeria. Several cases were declared in Senegal and Mali [1]. Isolated cases have been reported in the United Kingdom, United State and Spain(Briand [2]. As at 31st March, the world Health Organization (WHO) and respective governments have reported a total of 25,263 suspected cases and 10,477 deaths (see table 1).

Table 1: World Health Organization (WHO) Report as at 31st March 2015

COUNTRY	CASES	DEATH
Liberia	9,712	4,332
Sierra Leone	12,022	3,810
Guinea	3,494	2,320
Nigeria	20	8
Mali	8	6
United States	4	1
United Kingdom	1	0
Senegal	1	0
Spain	1	0
TOTAL	**25,263**	**10,477**

The incubation period of Ebola virus is 2 – 21 days, and the infectious period is 4 – 10 days [3]. The onset of the disease is characterized by tiredness, fever, and pain in the muscles and joints. Later symptoms may include server headache, nausea, sore throat, malaise, vomiting, bloody diarrhea and rash. Symptoms may even include loss of blood through internal and/or external bleeding [4]. Human-to-human transmission occurs only through direct contact with blood or bodily fluids from an infected person who has started exhibiting the symptoms or by contact with objects recently contaminated by an actively infected person [5].

Airborne transmission has not been proven. Deadly bodies are still infectious and so must be handled with great caution [6]. One study suggested that the virus can live up to 7 days in a diseased person. Semen and possibly other body fluids such as breast milk may be infectious in survivors for months [7].
Presently, there is no proven Ebola virus-specific treatment, however, there are measures that can be taken to improve a patients chances of survival [4].

One of the primary reasons for the spread of the virus is the poorly functioning health system in the part of Africa where the disease occurs. Other reasons include lack of adequate information on the virus and on the mode of spread. The risk of transmission is high among those caring for infected persons. Recommended measures when caring for an infected person include isolation through the proper use of gloves, masks, gown, boots and goggles as well as sterilizing equipment and surfaces [8].

2. METHODS

2.1 Model Description

The disease dynamics within the population shall be described using the standard SEIR model, in which the population is partitioned into four components or classes depending on the

epidemiological state of individuals in the population. The compartments are: the Susceptible, the Latent, the Infectious and the Recovered compartments.

The Susceptible compartment increases as new babies are born into the population as well as due to the fact that cured and recovered individual are again susceptible to the disease. The compartment decreases due to infection of some of the susceptible individuals who first become latently infected in that they have not yet started exhibiting the symptoms and so are not actively infectious. The compartment also decreases as a result of death from natural causes.

The latent compartment increases as a result of infection of some people in the susceptible class with the virus. It decreases due to the progression of some of the latently infected individuals to active or full infection, the recovery of some in this class due to treatment as well as due to natural death.
The Infectious compartment increase due to the progression of latently infected individuals to active infection and decreases due to the recovery of some individuals in this class, death from natural causes as well as death from Ebola-related causes.
The population of the Recovered components increases due to the recovery of people from both the latently infected and the infectious classes and decreases due to the re-entry of individuals in this class to the susceptible compartment as well as due to natural death.

2.2 Assumptions

The model is based on the following assumptions;

1) That the individuals that make up the population can be grouped into different compartments or groups based on their epidemiological state. In other word, the population is assumed to be heterogeneous.
2) That the population size in each compartment is differentiable with respect to time and that the epidemic process is deterministic. In other words, that the changes in population of a compartment can be calculated using only history to develop the model.
3) That the population mixes homogeneously. That is, all the susceptible individuals are equally likely to be infected by infectious individuals in case of contact.
4) That people in each compartment have equal natural death rate of μ.
5) That the only way of entry into the population is through newborn babies and the only way of exit is through death from natural causes or death from Ebola virus diseases related causes. In other word, that there are no immigration or emigration.
6) That the infection does not confer any immunity to the cured and recovered individuals and so they go back to the susceptible compartment.
7) That all newborns are previously uninfected by Ebola virus and therefore join only the susceptible class.
8) That there is presently no vaccination that provides immunity against the Ebola virus disease.
9) That infected individuals could be treated and cured of the virus.

2.3 Variables and Parameters

The variables and parameters used in this model are:

S(t): The population of susceptible individuals at time t
L(t): The population of latently infected individuals at time t
I(t): The population of infected individuals at time t
R(t): The population of individuals who have been treated and have recovered from the infection at time t.
α: The rate at which susceptible individuals become latently infected with the virus.
β: The rate at which latently infected individual become fully infected with the disease
π: The rate at which infected individuals recover from the disease.
κ: The rate at which recovered individuals return to the susceptible compartment.
μ: The natural mortality or death rate
d: The Ebola virus disease-induced mortality or death rate
Λ: The population of newborn babies entering the population.
N: The total population size.

The Ebola virus disease dynamics is schematically described in figure 1.

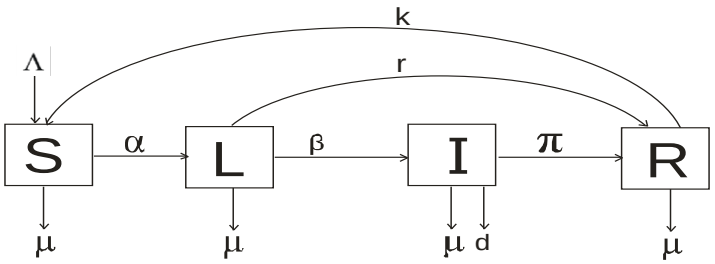

Figure 1: Schematic Presentation of the Model

Based on the assumptions and the inter-relationship between the variables and the parameters as described in the compartmental model in figure 1, the Ebola virus disease dynamics can be described by the following differential equations.

$$\frac{dS}{dt} = \Lambda - \alpha SI - \mu S + \kappa R \quad (1)$$

$$\frac{dL}{dt} = \alpha SI - (\mu + \beta)L - rL \quad (2)$$

$$\frac{dI}{dt} = \beta L - (\pi + \mu + d)I \quad (3)$$

$$\frac{dR}{dt} = \pi I - (\kappa + \mu)R + rL \quad (4)$$

$$N = S + L + I + R$$

3. RESULTS AND DISCUSSION

3.1 Equilibrum Point of the Model

At the equilibrium point we have:

$$\frac{dS}{dt} = \frac{dL}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

That is,

$$\Lambda - \alpha SI - \mu S + \kappa R = 0 \quad (5)$$

$$\alpha SI - (\mu + \beta + r)L = 0 \quad (6)$$

$$\beta L - (\pi + \mu + d)I = 0 \quad (7)$$

$$\pi I + rL - (\kappa + \mu)R = 0 \quad (8)$$

3.2 The Endemic Equilibrium State or Ebola Virus Disease

Let the endemic equilibrium of Ebola be $E^*(S^*, L^*, I^*, R^*)$. The endemic equilibrium state of the Ebola virus disease is the state where the disease is persistent in the population. For the disease to be persistent, we must have at equilibrium:

$$S^* = L^* = I^* = R^* \neq 0$$

Therefore, we solve equations (5) – (8) simultaneously for $S^*, L^*, I^* and\ R^*$.

From equ. (6), $\alpha SI = (\mu + \beta + r)L$

$$\therefore L = \frac{\alpha SI}{(\mu+\beta+r)} \quad (9)$$

From equ. (7), $\beta L = (\pi + \mu + d)I$

$$L = \frac{(\pi+\mu+d)I}{\beta} \quad (10)$$

Equating equ. (9) and (10) we have

$$\frac{\alpha SI}{(\mu+\beta+r)} = \frac{(\pi+\mu+d)I}{\beta}$$

$$\left[\frac{\alpha S}{(\mu+\beta+r)} - \frac{(\pi+\mu+d)}{\beta}\right] I = 0$$

Since I ≠ 0, we have

$$\frac{\alpha S}{(\mu+\beta+r)} - \frac{(\pi+\mu+d)}{\beta} = 0$$

$$\Longrightarrow S^* = \frac{(\mu+\beta+r)(\pi+\mu+d)}{\alpha\beta} \quad (11)$$

Substitute equ.(11) into equ.(5) to have

$$\Lambda - \frac{\alpha((\mu+\beta+r)(\pi+\mu+d)I}{\alpha\beta} - \frac{\mu(\mu+\beta+r)(\pi+\mu+d)}{\alpha\beta} + \kappa R = 0$$

$$\alpha\beta\,\Lambda - (\mu+\beta+r)(\pi+\mu+d)I - \mu((\mu+\beta+r)(\pi+\mu+d) + \alpha\beta\kappa R = 0$$

$$-(\mu+\beta+r)(\pi+\mu+d)I + \alpha\beta\kappa R = \mu((\mu+\beta+r)(\pi+\mu+d) - \alpha\beta\,\Lambda \quad (12)$$

Substitute equ. (10) into equ.(8) to have

$$\pi I + \frac{r(\pi+\mu+d)I}{\beta} - (\kappa+\mu)R = 0$$

$$[\beta\pi + r(\pi+\mu+d)]I - \beta(\kappa+\mu)R = 0 \quad (13)$$

Solving equ.(12) and (13) simultaneously we have

Equ.(12) x $(\kappa+\mu)$ gives

$$-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)I+\alpha\beta\kappa(\kappa+\mu)R=\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu) \quad (14)$$

Equ.(13) x $\alpha\kappa$ gives

$$\alpha\kappa[\beta\pi+r(\pi+\mu+d)]I-\alpha\beta\kappa(\kappa+\mu)R=0 \quad (15)$$

Adding equ. (14) and equ.(15) gives

$$\alpha\kappa[\beta\pi+r(\pi+\mu+d)]I-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)I=\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)$$

$$\therefore\ I^*=\frac{\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)} \quad (16)$$

Substitute equ.(16) into equ.(10) to have

$$L^*=\frac{(\pi+\mu+d)}{\beta}\left[\frac{\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)}\right] \quad (17)$$

From equ.(8) ,$R(\kappa+\mu)=\pi I+rL$

$$\Longrightarrow R=\frac{\pi I+rL}{(\kappa+\mu)} \quad (18)$$

Substituting equ.(16) and equ.(17) into equ (18) gives

$$R=\frac{\frac{\pi\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)}+\frac{r(\pi+\mu+d)}{\beta}\left[\frac{\mu(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)}\right]}{(\kappa+\mu)}$$

$$\therefore R^*=\frac{\pi\mu(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)}+\frac{r(\pi+\mu+d)}{\beta}\left[\frac{\mu(\mu+\beta+r)(\pi+\mu+d)-\alpha\Lambda\beta(\kappa+\mu)}{\alpha\kappa[\beta\pi+r(\pi+\mu+d)]-(\kappa+\mu)(\mu+\beta+r)(\pi+\mu+d)}\right]$$

……equ.(19)

3.3 The Stability Analysis of the Endemic Equilibrium State of Ebola Virus Disease.

The Jacobian matrix J of the system of equations is

$$J=\begin{pmatrix} -(\alpha I+\mu) & 0 & -\alpha S & \kappa \\ \alpha I & -(\mu+\beta+r) & \alpha S & 0 \\ 0 & \beta & -(\pi+\mu+d) & 0 \\ 0 & r & \pi & -(\kappa+\mu) \end{pmatrix}$$

The values of S, L, I, R at the endemic equilibrium state is given by equs. (11), (17), (16) and (19) respectively. At the endemic equilibrium point, E^*, the Jacobian matrix is

$$J^*=\begin{pmatrix} -(\alpha I^*+\mu) & 0 & -\alpha S^* & \kappa \\ \alpha I^* & -(\mu+\beta+r) & \alpha S^* & 0 \\ 0 & \beta & -(\pi+\mu+d) & 0 \\ 0 & r & \pi & -(\kappa+\mu) \end{pmatrix}$$

If λ_i are the eigen values of the Jacobian matrix, then the characteristic equation is $[J^* - \lambda I] = 0$

That is,

$$[J^* - \lambda I] = \begin{vmatrix} -(\alpha I^* + \mu) - \lambda & 0 & -\alpha S^* & \kappa \\ \alpha I^* & -(\mu + \beta + r) - \lambda & \alpha S^* & 0 \\ 0 & \beta & -(\pi + \mu + d) - \lambda & 0 \\ 0 & r & \pi & -(\kappa + \mu) - \lambda \end{vmatrix} = 0$$

The Routh-Hurwitz necessary and sufficient condition for the system to be asymptotically stable is that all the roots (eigen values) of the characteristic equation must have negative real part. The computations to get the four roots (eigen values) of the characteristic equation is very complex and so renders this approach inappropriate [9]. described the use of the basic reproduction number, R_0 in analyzing the stability of the endemic equilibrium point.

The basic reproduction number, R_0, is defined by [10] and [11] as the average number of secondary infections caused by an infectious individual during his/her life as an infectious person. When $R_0 > 1$, the system has a unique endemic equilibrium that is globally asymptotically stable and when $R_0 < 1$ the system has an unstable endemic equilibrium [9]. This technique shall be adopted in the analysis of the stability of the endemic equilibrium point of Ebola virus disease.

The values of R_0 in each of the sub-epidemics is determined by the product of three components, namely:

1. The average number of infections that one infectious case causes per unit time.
2. The average time that an individual remains infectious.
3. The probability that a latent case will develop into an infectious case [12].

Adopting this definition for our mode we have that the R_0 for Ebola is given by:

$$R_o = \left(\frac{\Lambda + \kappa}{\mu + \alpha}\right)\left(\frac{\alpha}{\mu + \beta + r}\right)\left(\frac{\beta}{\mu + d + \pi}\right)$$

For the endemic equilibrium point to be globally asymptotically stable we must have $R_o > 1$. That is,

$$R_o = \left(\frac{\Lambda + \kappa}{\mu + \alpha}\right)\left(\frac{\alpha}{\mu + \beta + r}\right)\left(\frac{\beta}{\mu + d + \pi}\right) > 1$$

$$= \left(\frac{\Lambda + \kappa}{\mu + \alpha}\right)\left(\frac{\alpha}{\mu + \beta + r}\right)\left(\frac{\beta}{\mu + d + \pi}\right) > 1$$

$$\alpha\beta(\Lambda + \kappa) > (\mu + \alpha)(\mu + \beta + r)(\mu + d + \pi)$$

$$\alpha\beta > \frac{(\mu+\alpha)(\mu+\beta+r)(\mu+d+\pi)}{(\Lambda+\kappa)} \quad (20)$$

If the virus is to be controlled, we must ensure that equilibrium state is unstable. That is, we must ensure that

$$\alpha\beta < \frac{(\mu+\alpha)(\mu+\beta+r)(\mu+d+\pi)}{(\Lambda+\kappa)} \quad (21)$$

Equation (21) gives the necessary condition for the endemic equilibrium state of Ebola to be unstable. This means that Ebola virus disease can be controlled or even be eradicated if the total break down of the susceptible and latent classes is always less than the total removal rates from the susceptible, the latent and the infectious classes.

4. CONCLUSION

A mathematical model for the control of the spread of the Ebola virus disease in any society was developed based on some assumptions and on the standard SEIR model, were the entire population is partitioned into compartments or classes, based on the epidemiological state of individuals within the population. The disease transmission dynamics within the population was described by a system of differential equations. The endemic equilibrium state of the model was established and its stability analysis carried out using the basic Reproduction Number. The condition under which TB can effectively be controlled was established.

5. Recommendations

The condition for the control or total eradication of the Ebola virus disease can be met if the following recommendations are considered:

1. People should be educated on the mode of transmission and on the symptoms of the diseases.
2. The conditions that promote rapid spread of the virus such as illiteracy, lack of adequate medical facilities, overcrowded accommodation etc. should be taken care of.
3. People who are infected should be encouraged to voluntarily report to designated health centers for immediate attention.
4. There should be more training of medical staff to specially handle Ebola virus disease.
5. There should be insurance policy for all medical staff handling Ebola virus disease to encourage them to be very serious and committed on handling infected individuals.
6. There should not be stigmatization of people infected by the disease or people fully cured of the disease.
7. People should freely submit themselves for Ebola tests and those found to be infected with the virus should co-operate with medical personnel.
8. Infected individuals should promptly be quarantined.
9. There should be more international co-operation to prevent cross-border transmission of the disease.
10. There should be more effort at producing vaccines and drugs for the virus by national and international organizations.

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